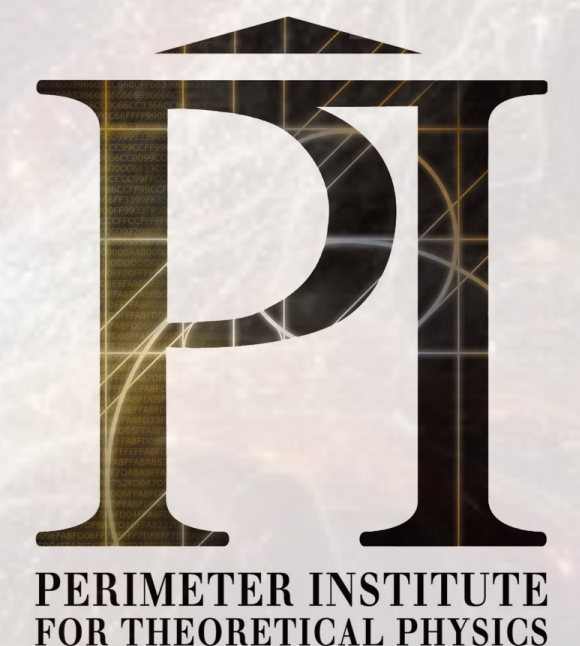


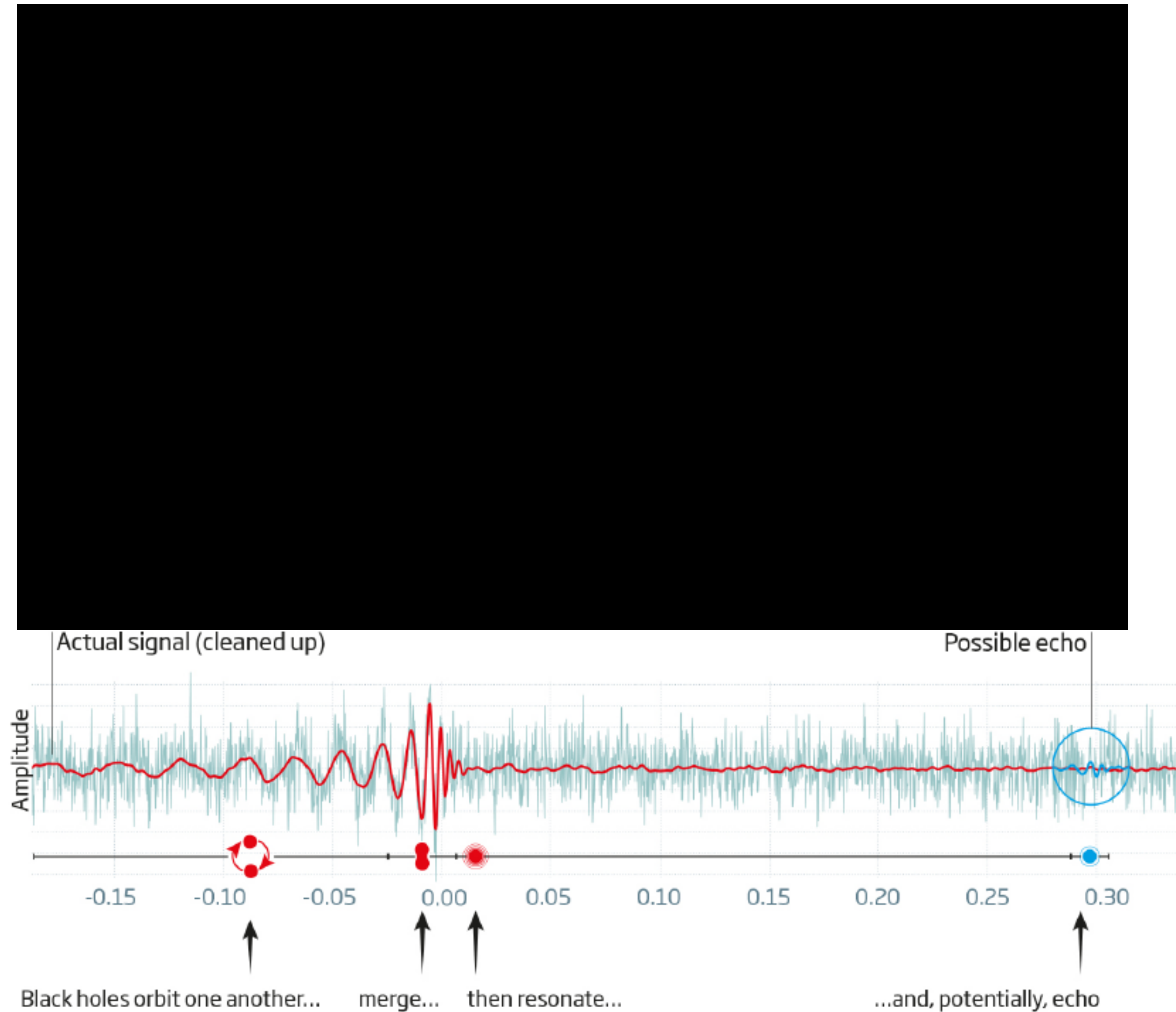
The 1st Tsinghua Relativity Workshop
24 July, 2024

Imaging Quantum Black Holes & (Towards) Optimal Estimators of Stochastic Gravitational Wave Background

Niayesh Afshordi

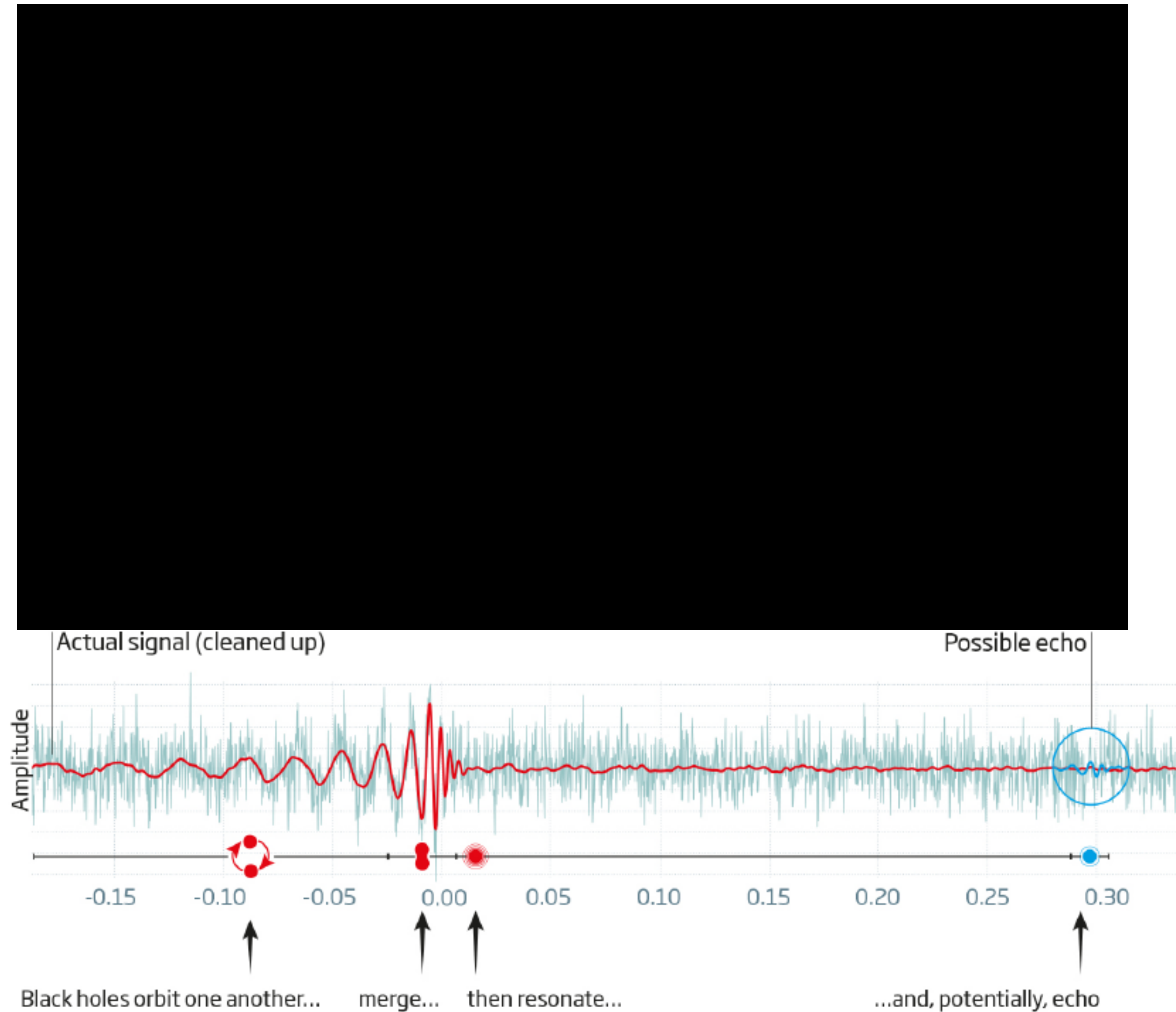


Gravitational Waves from firewalls?

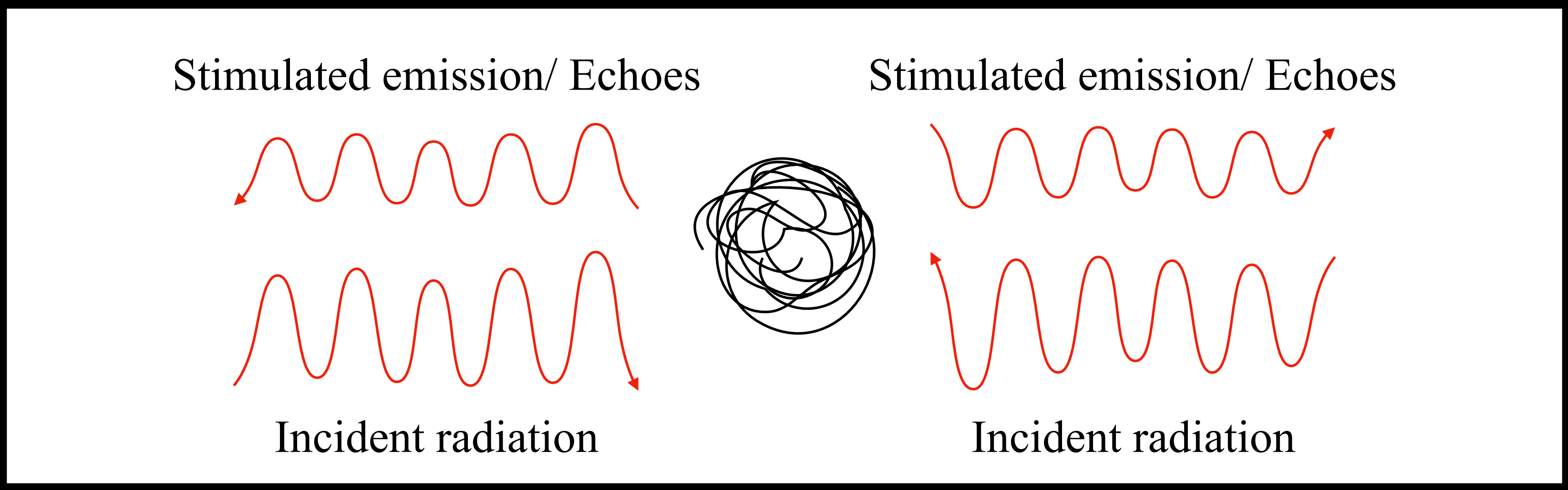
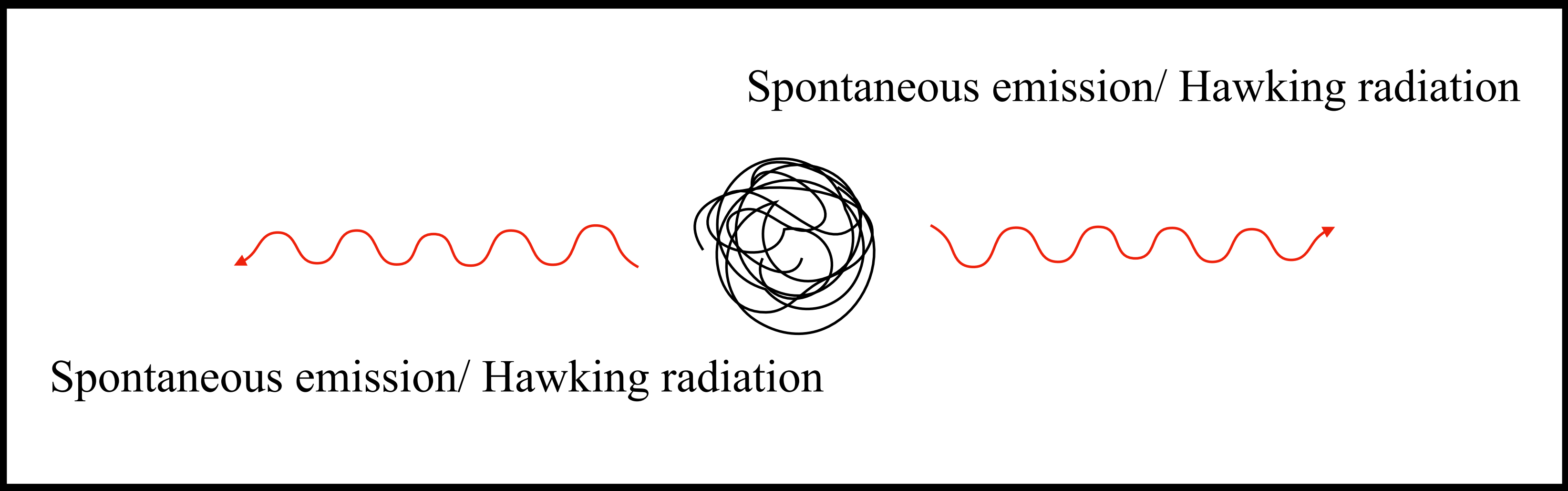


SOURCE: doi.org/bchwr; NIAYESH AFSHORDI AND JAHED ABEDI

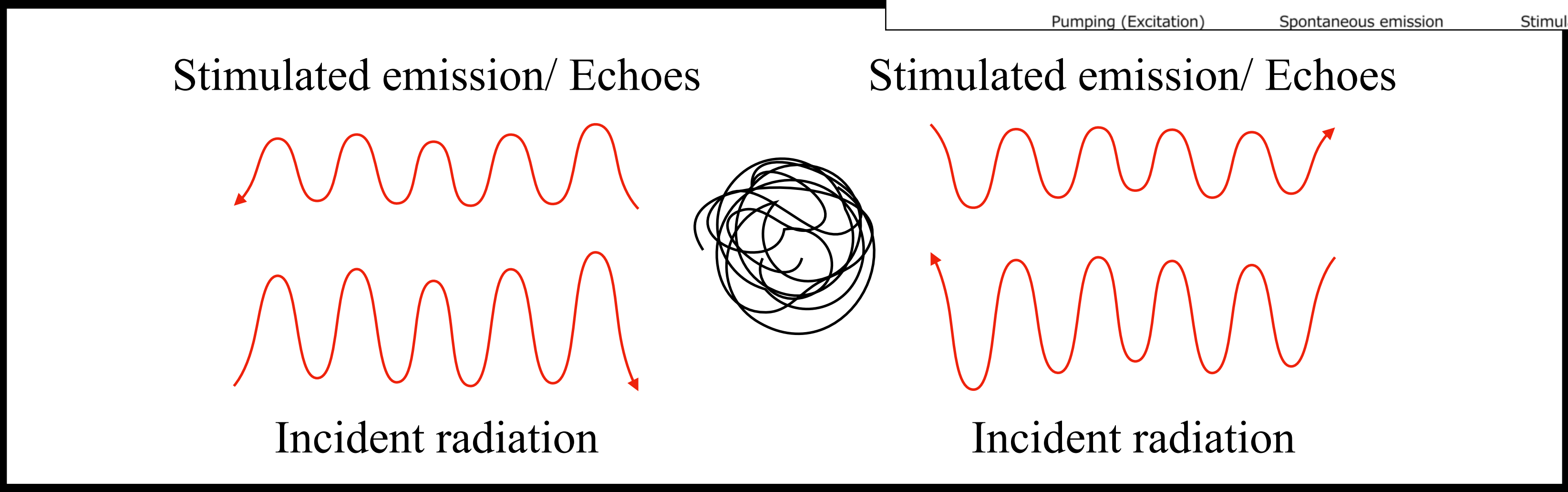
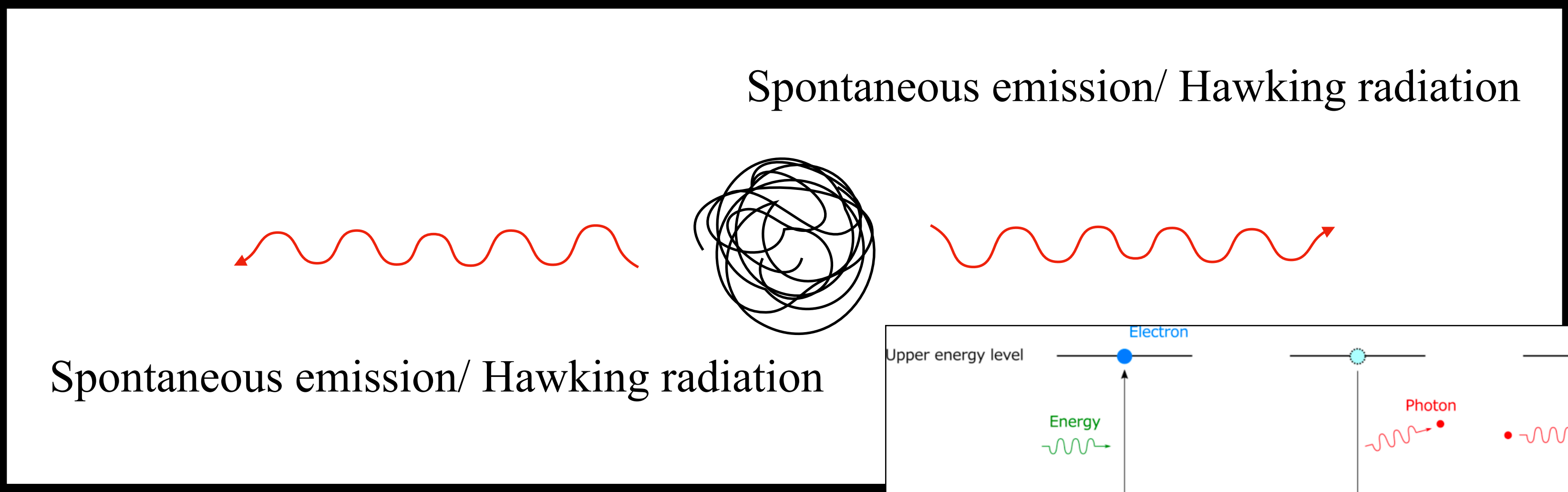
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$$R = \exp \left(-\frac{\hbar\omega}{kT_H} \right)$$



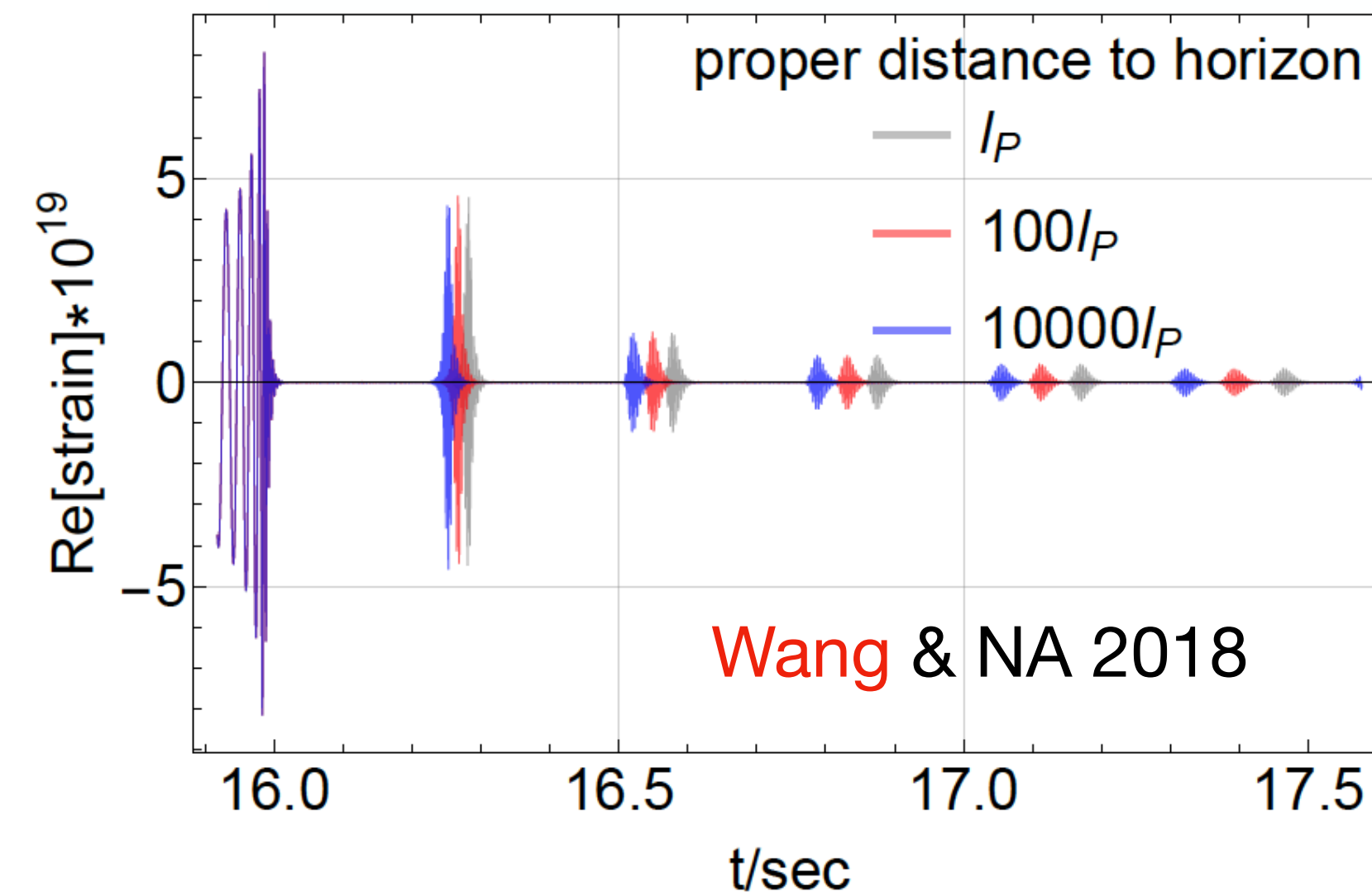
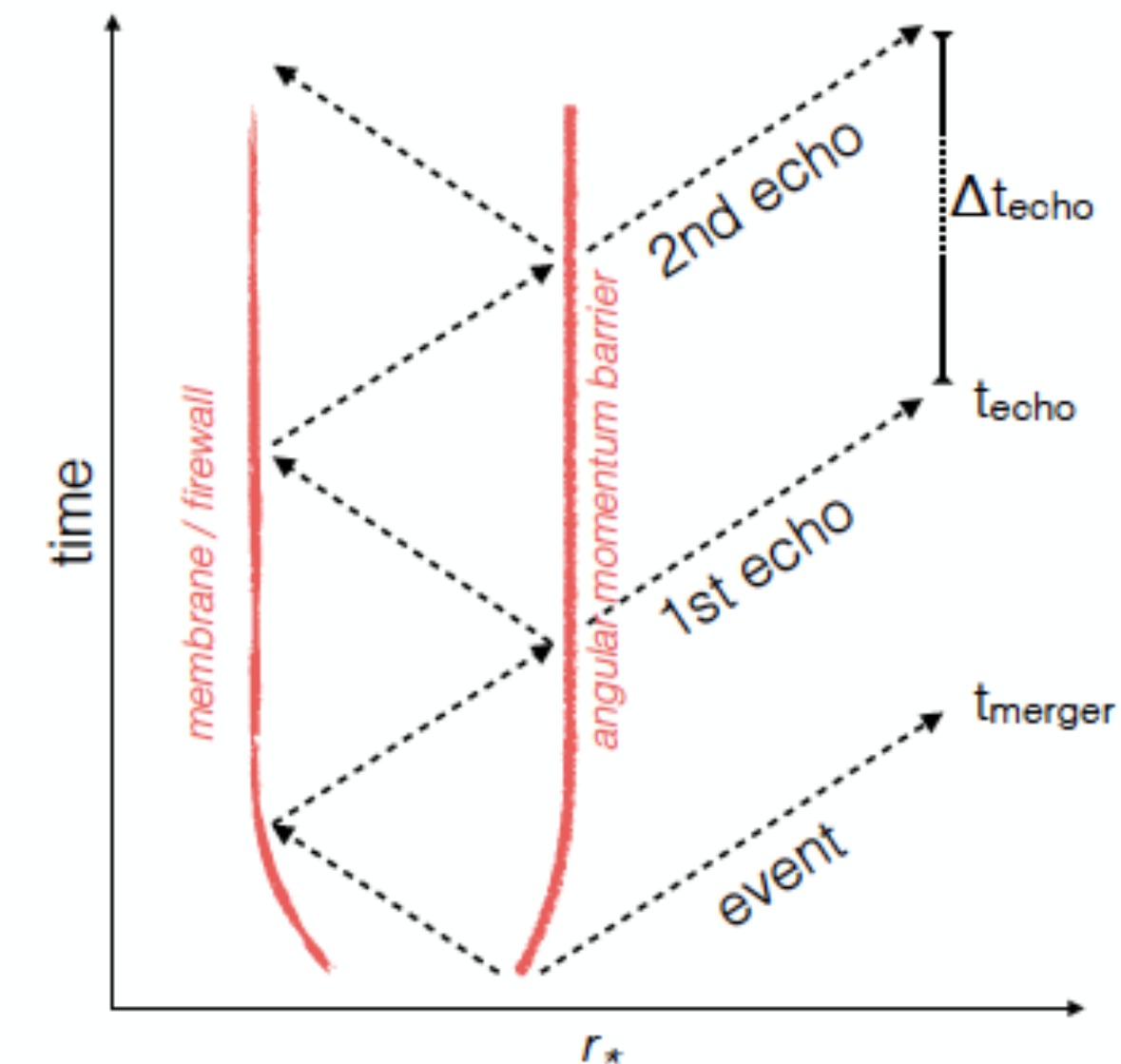
$$R = \exp \left(-\frac{\hbar\omega}{kT_H} \right)$$

From Planck to LIGO!

- Quantum effects within a Planck length of horizon
- “Echoes” in LIGO observations
(Cardoso, Franzin & Pani 16)

$$\Delta t_{\text{echo}} \simeq \frac{4GM_{\text{BH}}}{c^3} \left(1 + \frac{1}{\sqrt{1-a^2}} \right) \times \ln \left(\frac{GM_{\text{BH}}}{c^2 \ell_{\text{QG}}} \right)$$

- *Stimulated Hawking Radiation*
(Oshita, Wang & NA 2020)

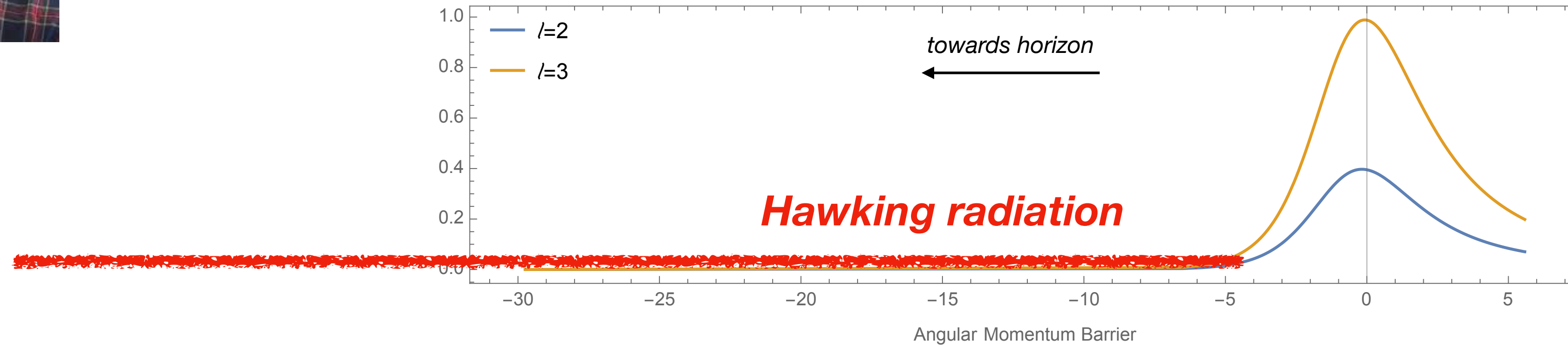


Finite Entropy of Hawking radiation

➡ **Echoes** (*Oshita & NA 2023*)



- Infinite Entropy & No Echoes: $t_{\text{echo}} \rightarrow \infty$!

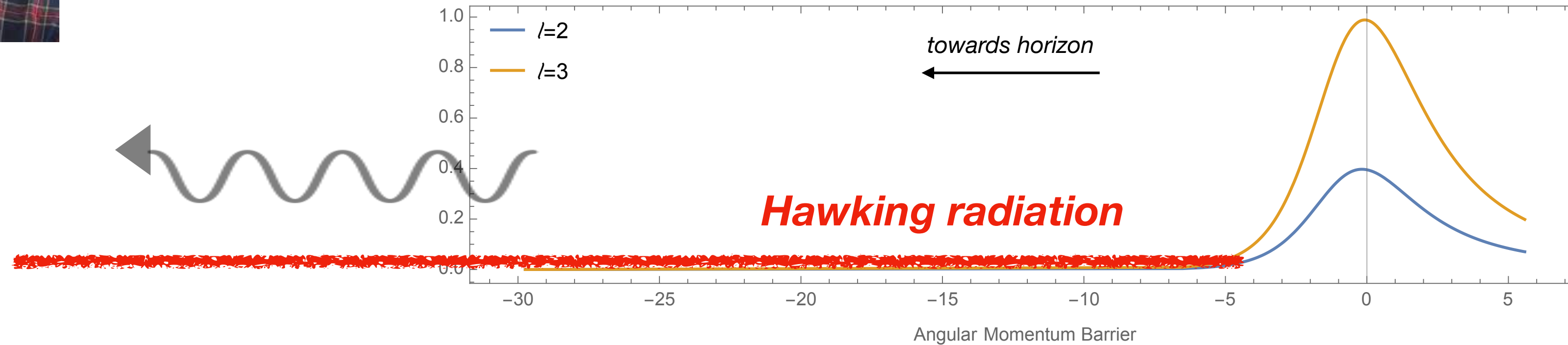


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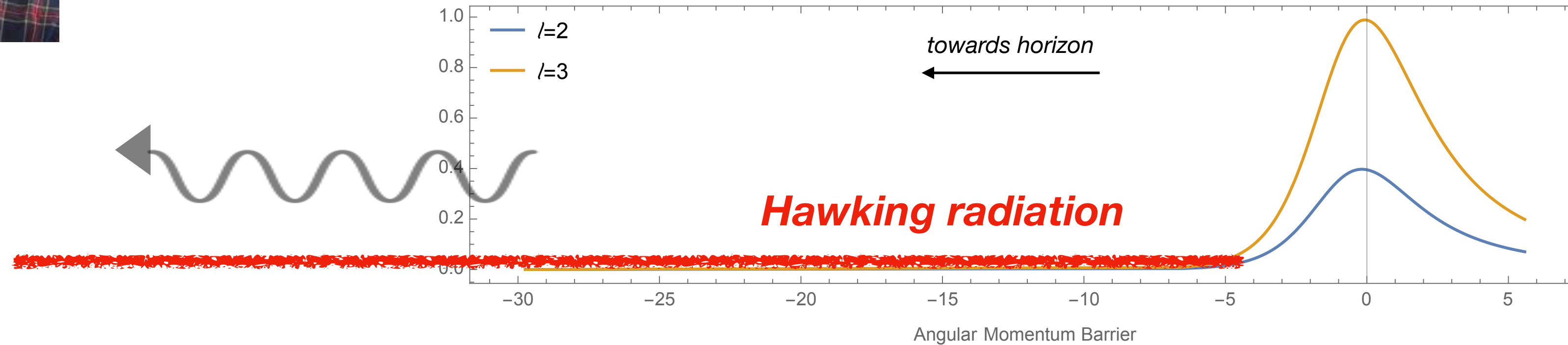


Finite Entropy of Hawking radiation

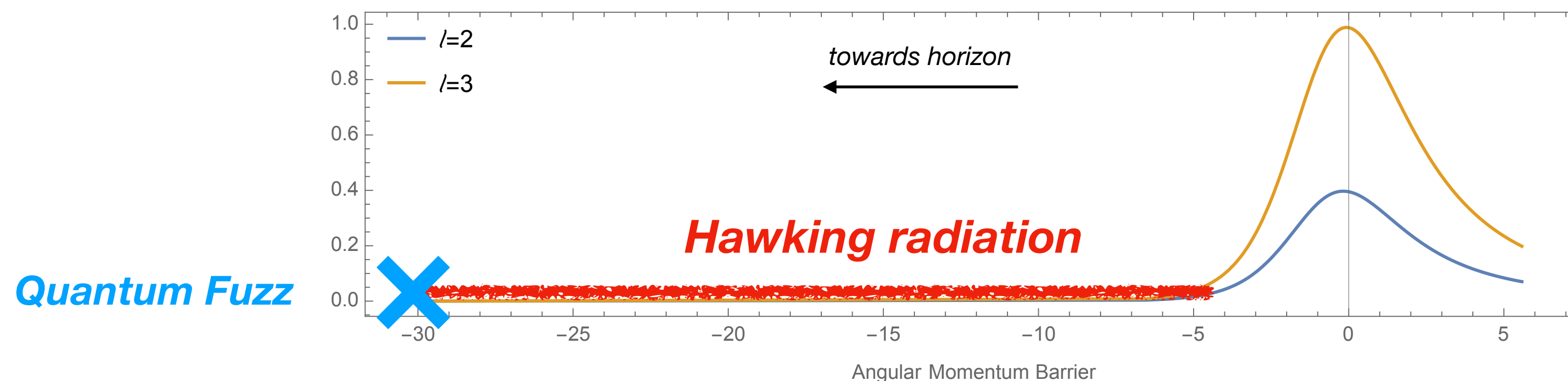
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- Infinite Entropy & No Echoes: $t_{\text{echo}} \rightarrow \infty$!



- Finite Entropy & Echoes: $t_{\text{echo}} = \frac{\ln [\text{Entropy}/\ell(\ell + 1)]}{2\pi \times \text{Hawking Temperature}}$

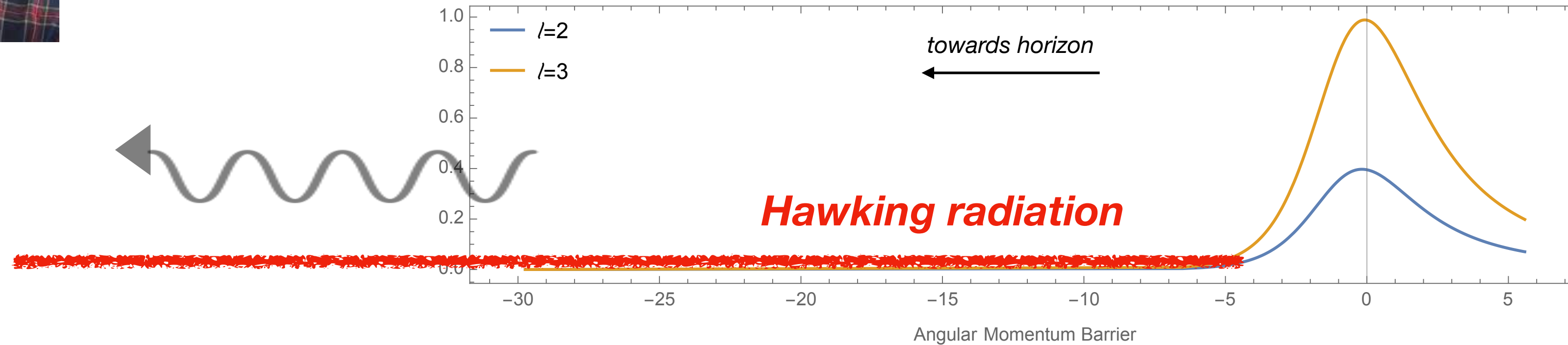


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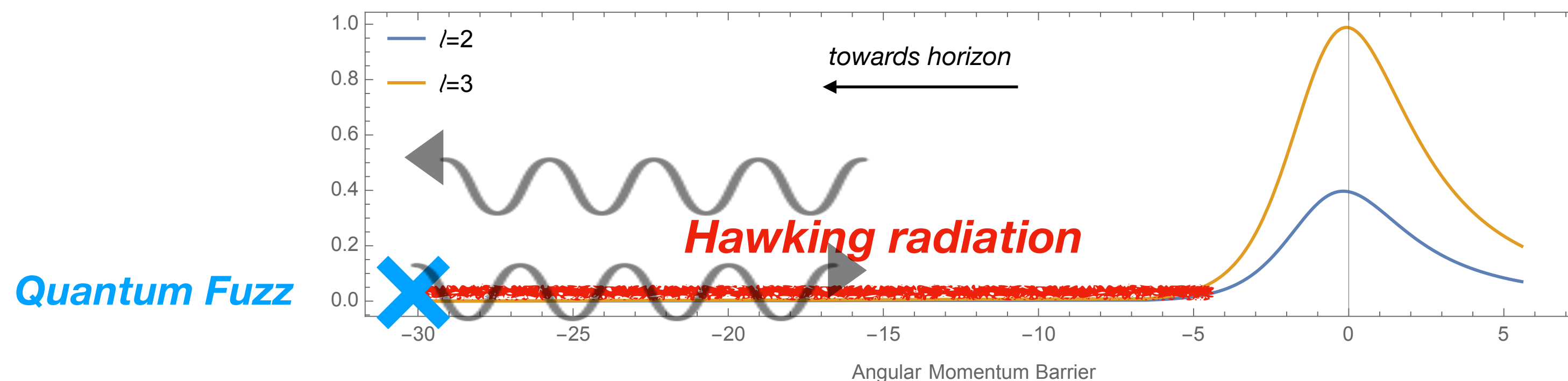
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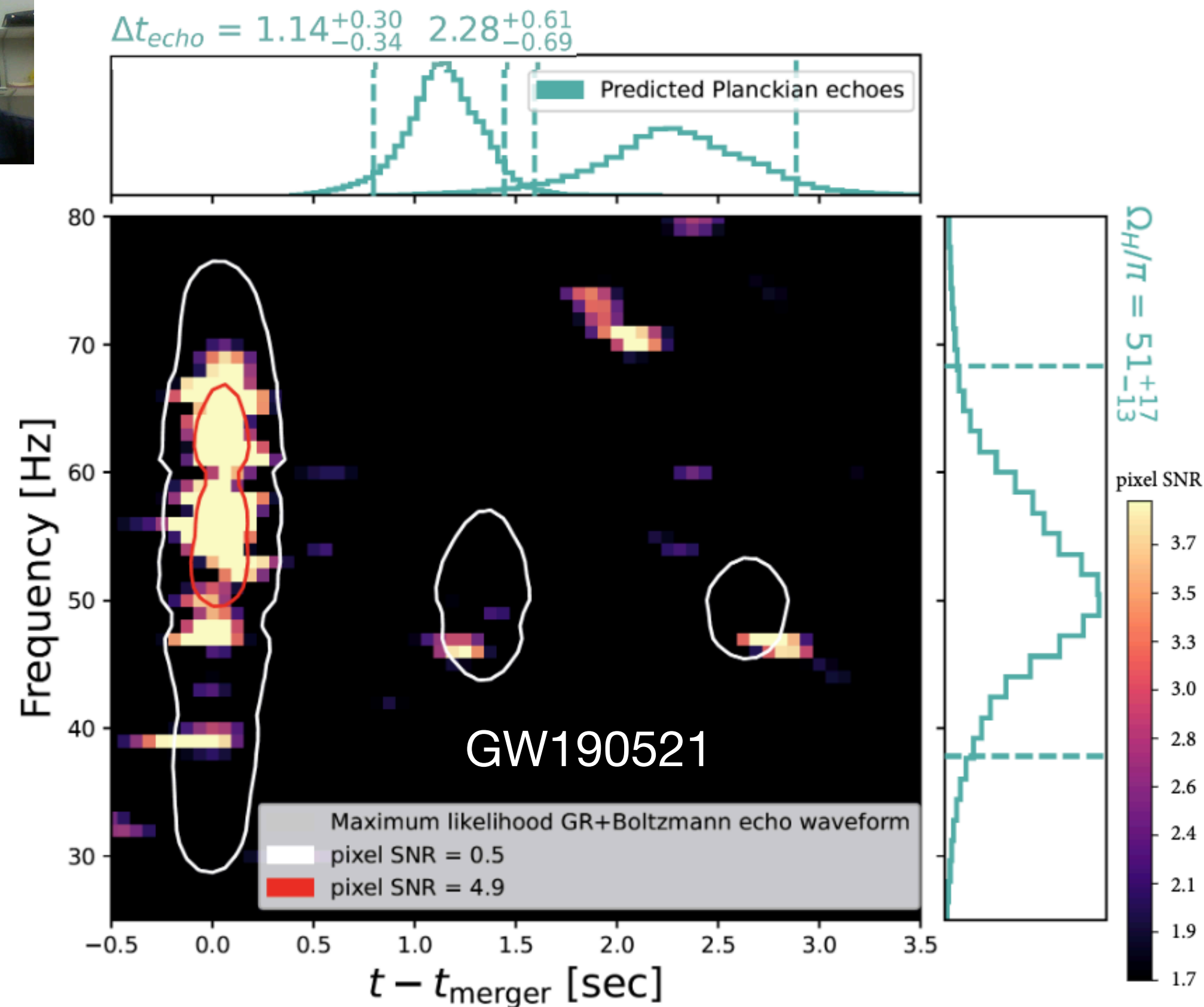


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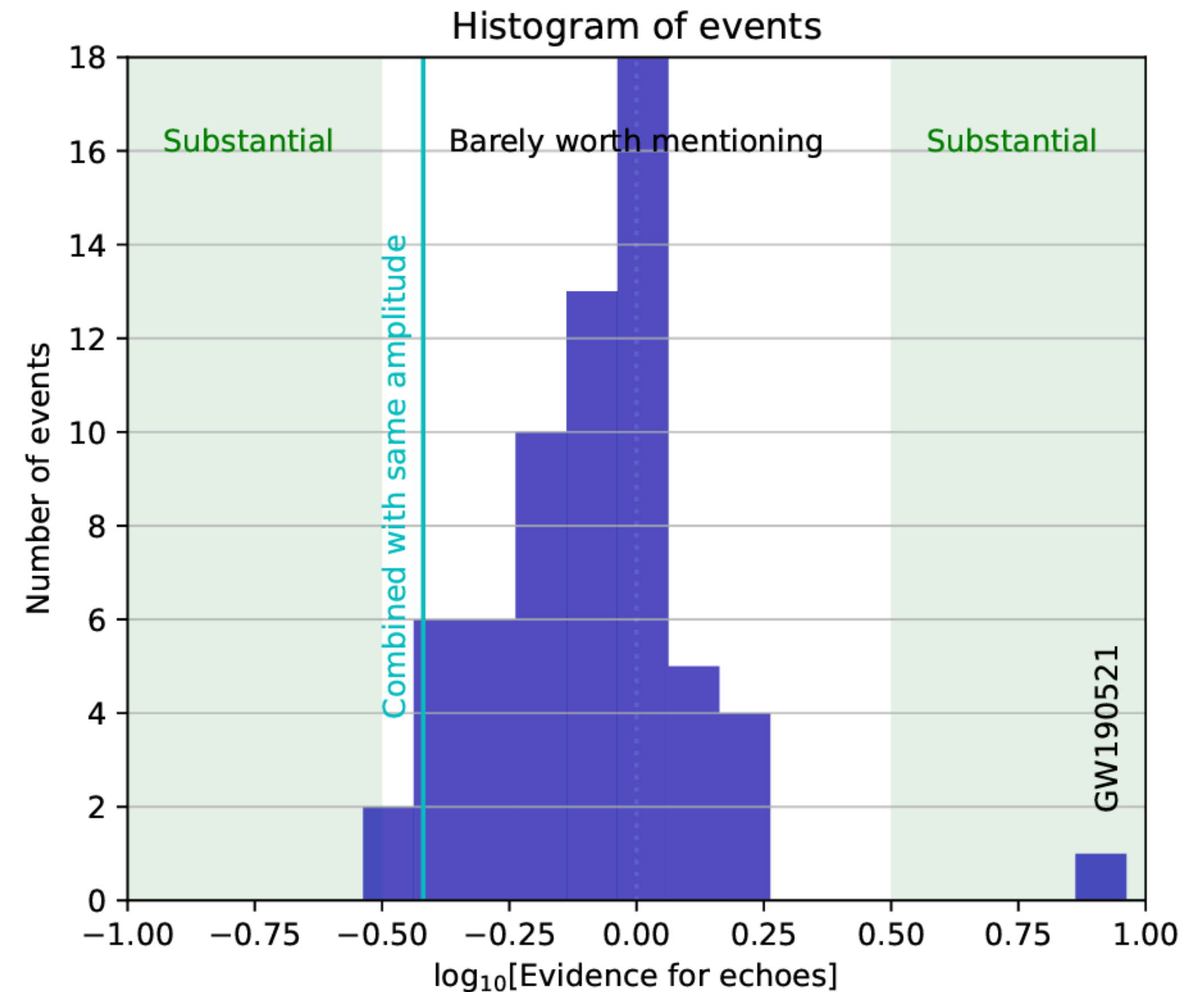


Are there echoes in LIGO data?

- Abedi, Longo & NA 2023

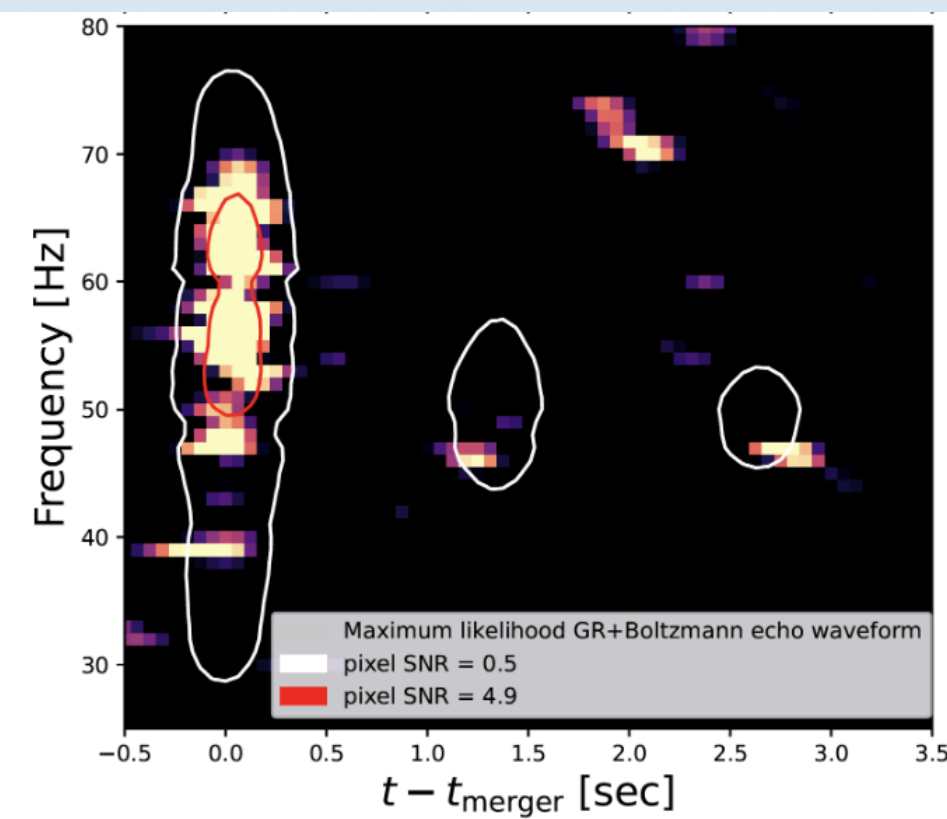


Abedi 2023



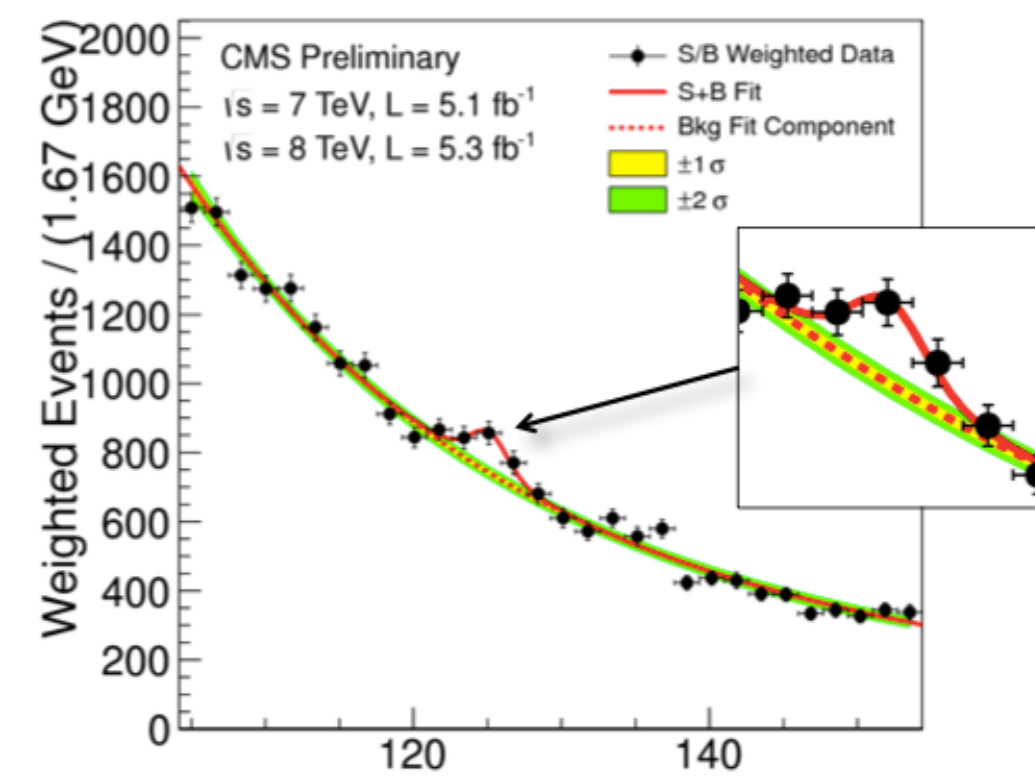
Echoes to *LIGO*

- Unitarity
- (Perturbative) Effective Field Theory
- Holographic Entropy ↔
Diffeomorphism sym.



Higgs to *LHC*

- Unitarity
- (Perturbative) Effective Field Theory
- Gauge Symmetries of Standard Model



Future is bright!

- Echoes in “numerical relativity”?
- Hybrid PPN+Teukolsky with matching (Qingwen Wang’s PhD thesis)

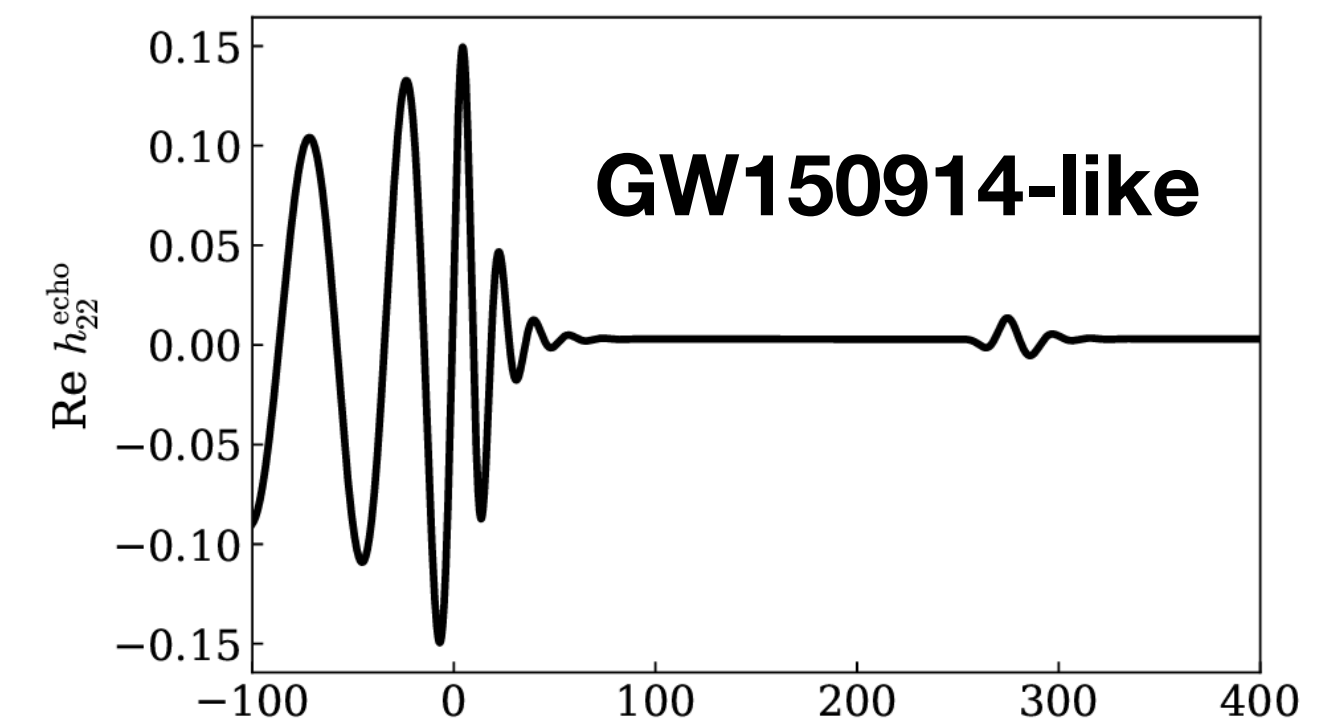
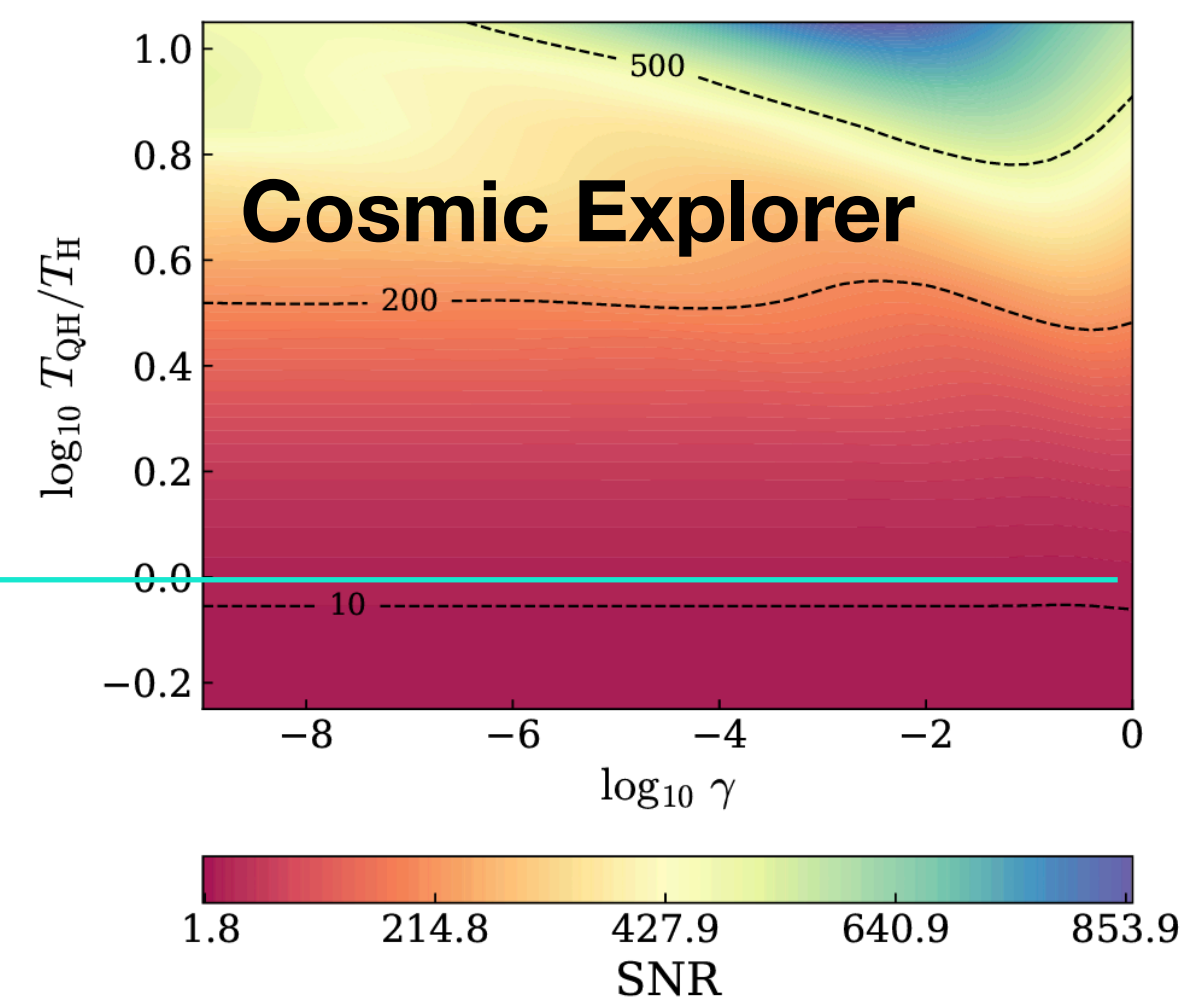
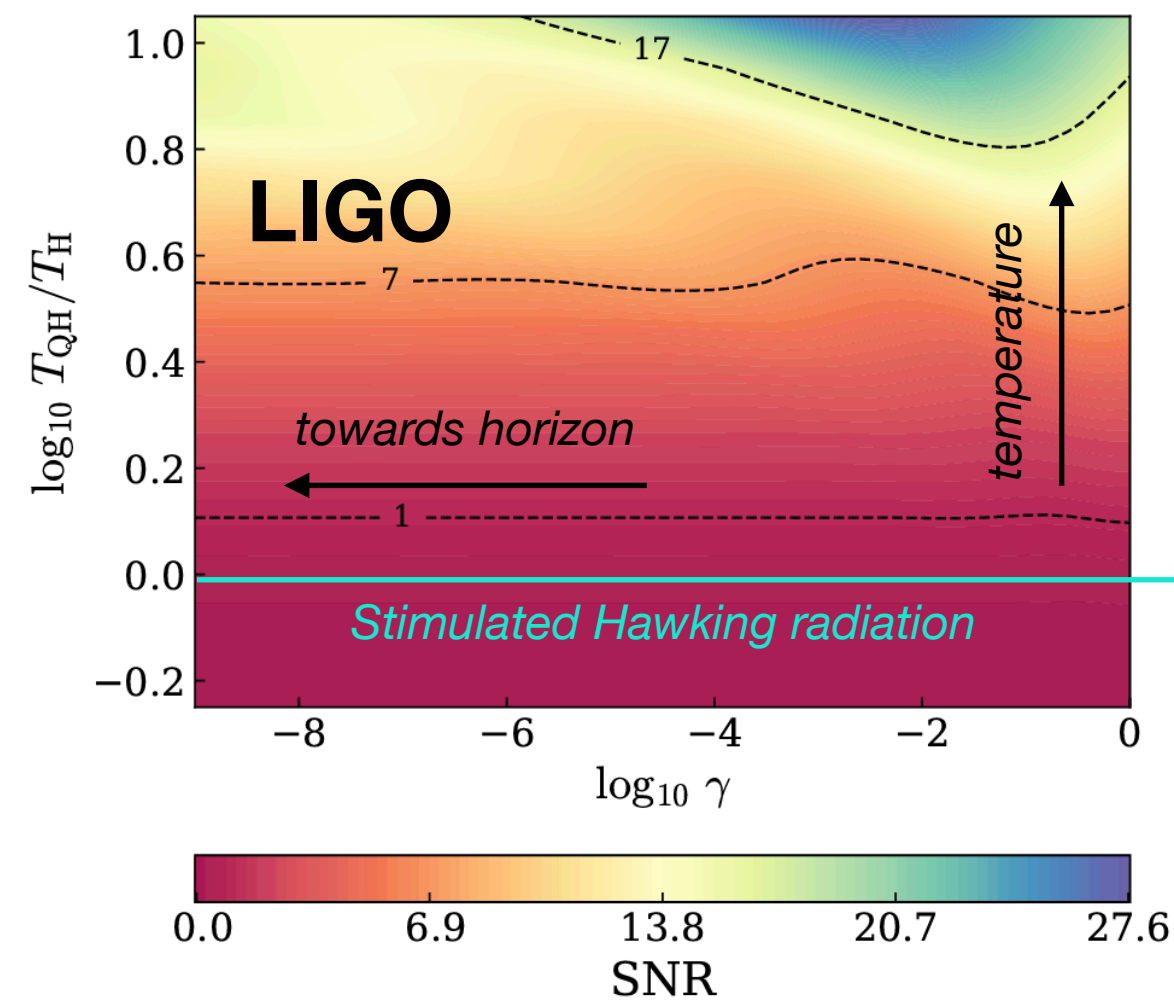
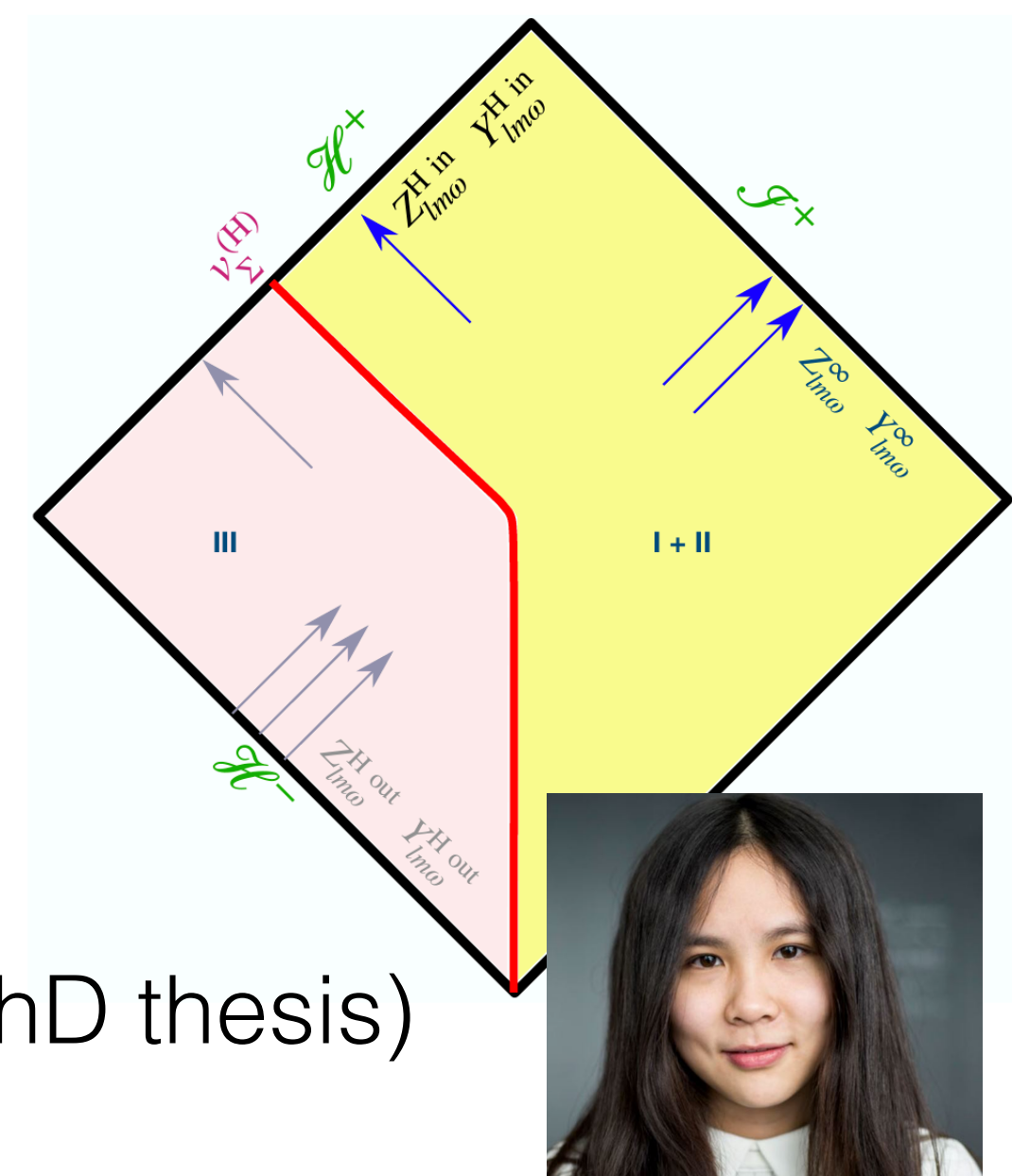


FIG. 11. The echo emitted by SXS:BBH:0207, following the main GW. Here we set $v_{\Sigma}^{(H)} = -13$, $\Delta v = 2/\kappa = 8$, $\gamma = 10^{-15}$, and $T_{QH} = T_H$.

arXiv > gr-qc > arXiv:2203.03174

General Relativity and Quantum Cosmology

[Submitted on 7 Mar 2022]

Gravitational-wave echoes from numerical-relativity waveforms via space-time construction near merging compact objects

Sizheng Ma, Qingwen Wang, Nils Deppe, Nils L. Fischer, François Hébert, Lawrence E. Kidder, Jordan Moxon, William Throwe, Mark A. Scheel, Yanbei Chen

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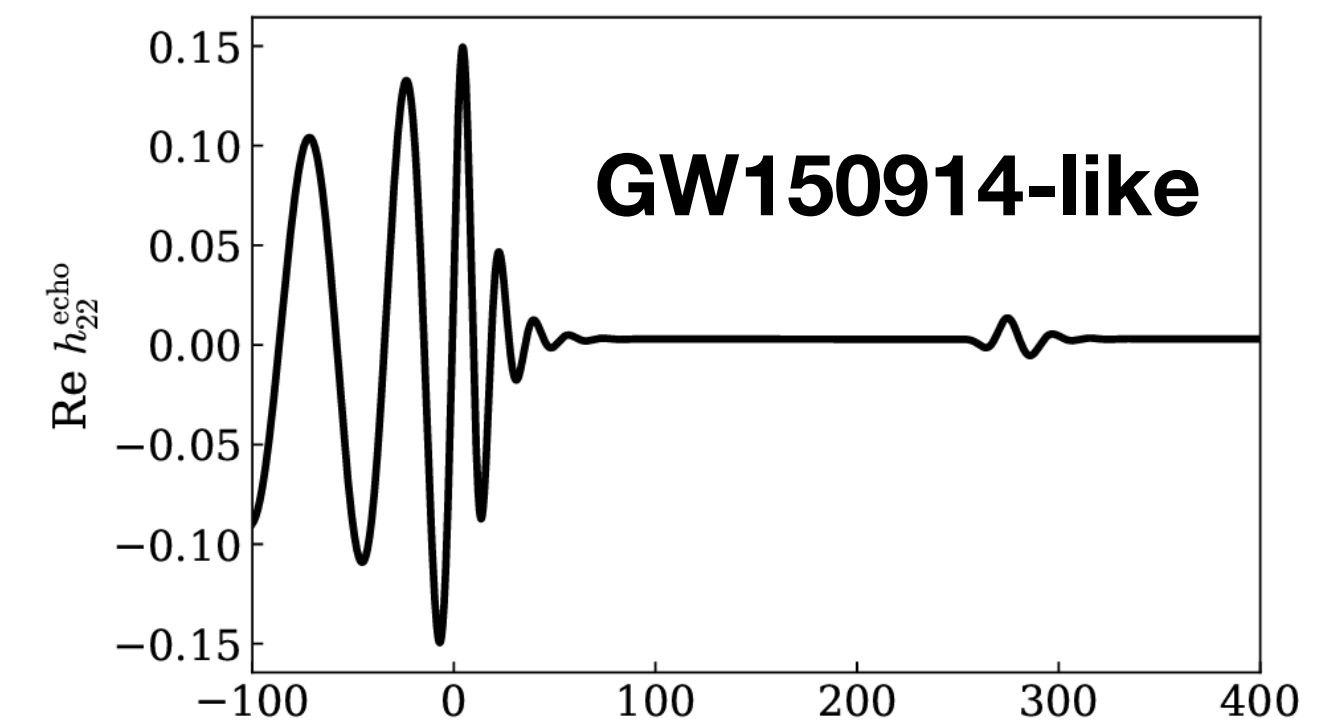
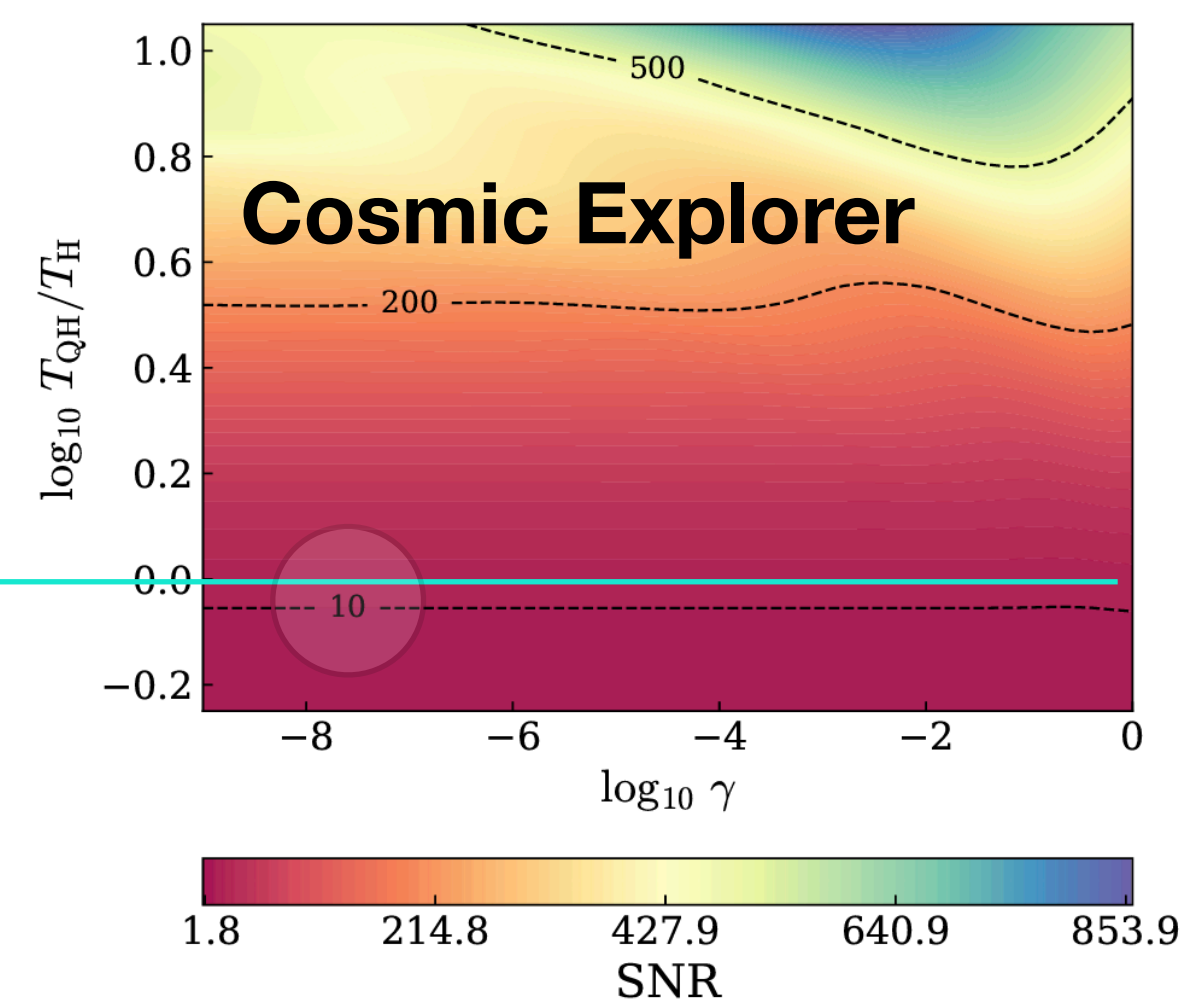
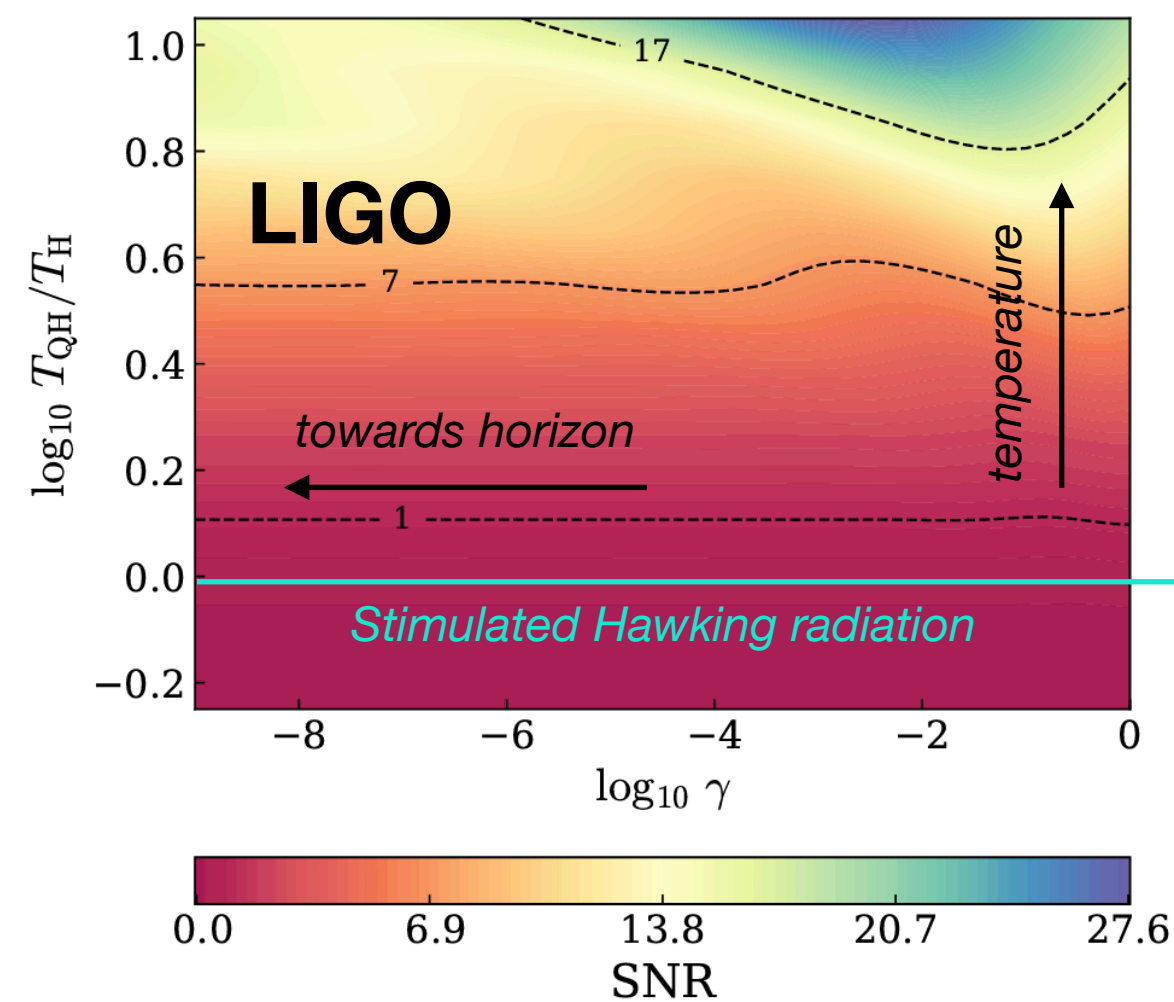
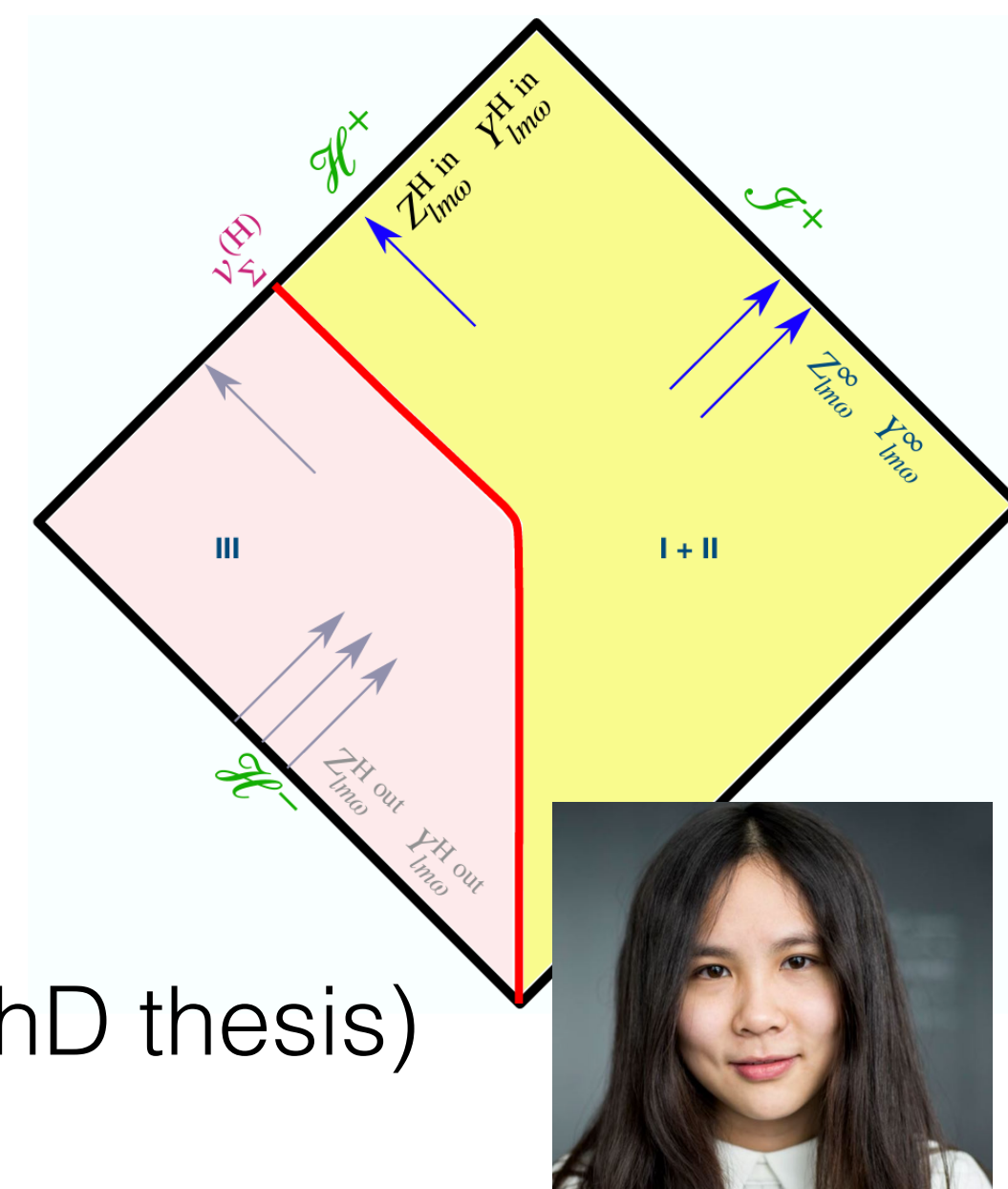


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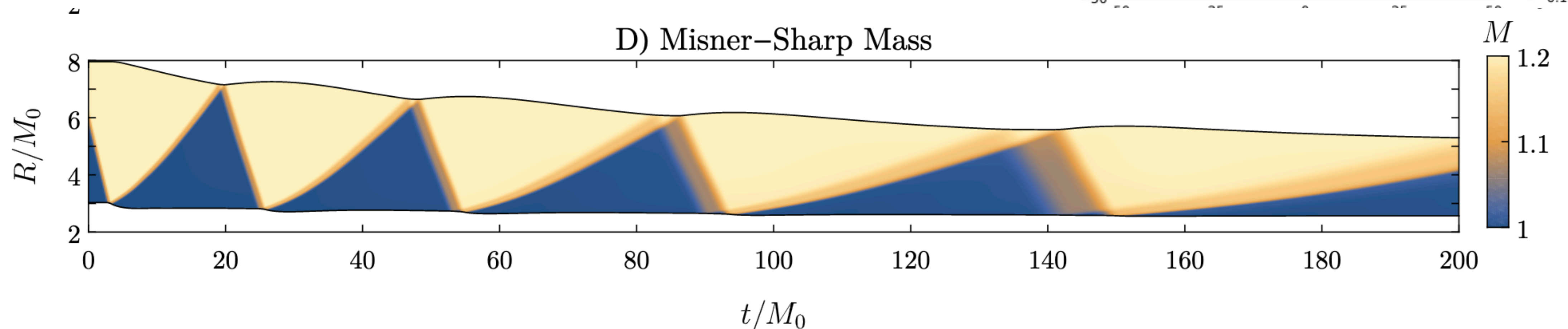
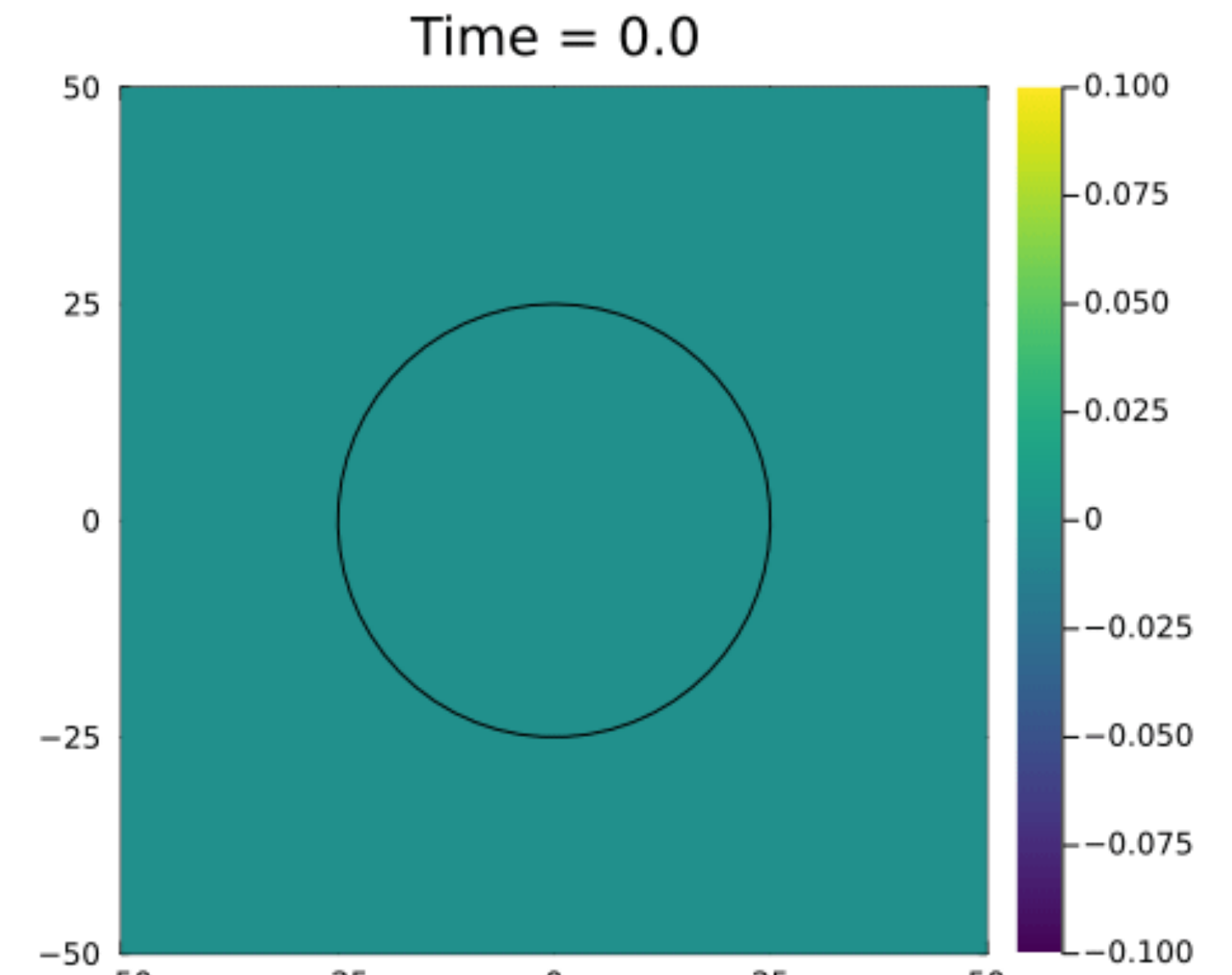
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Black Hole Echoes in Numerical Relativity

We are starting to simulate general relativistic spacetimes with “quantum” physical boundaries

Dailey, NA & Schnetter 2023 (+ in prep.)

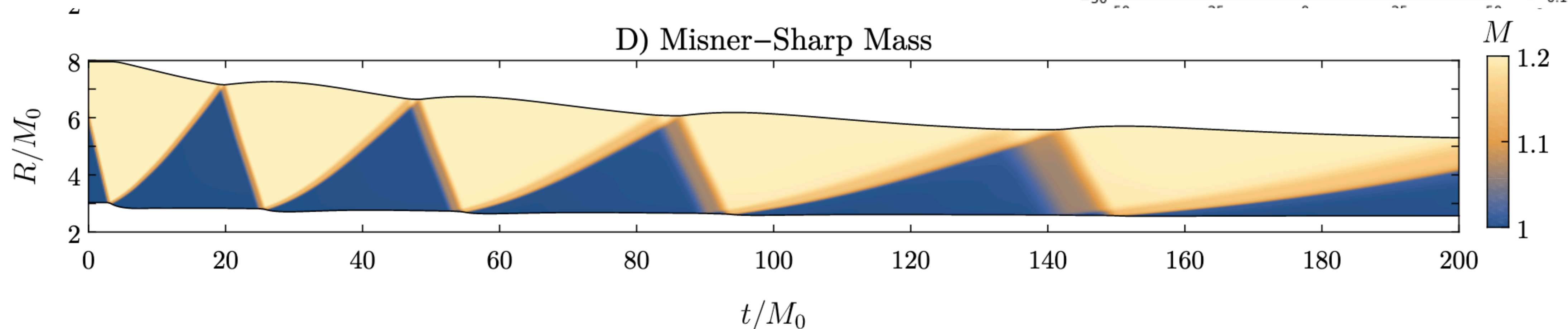
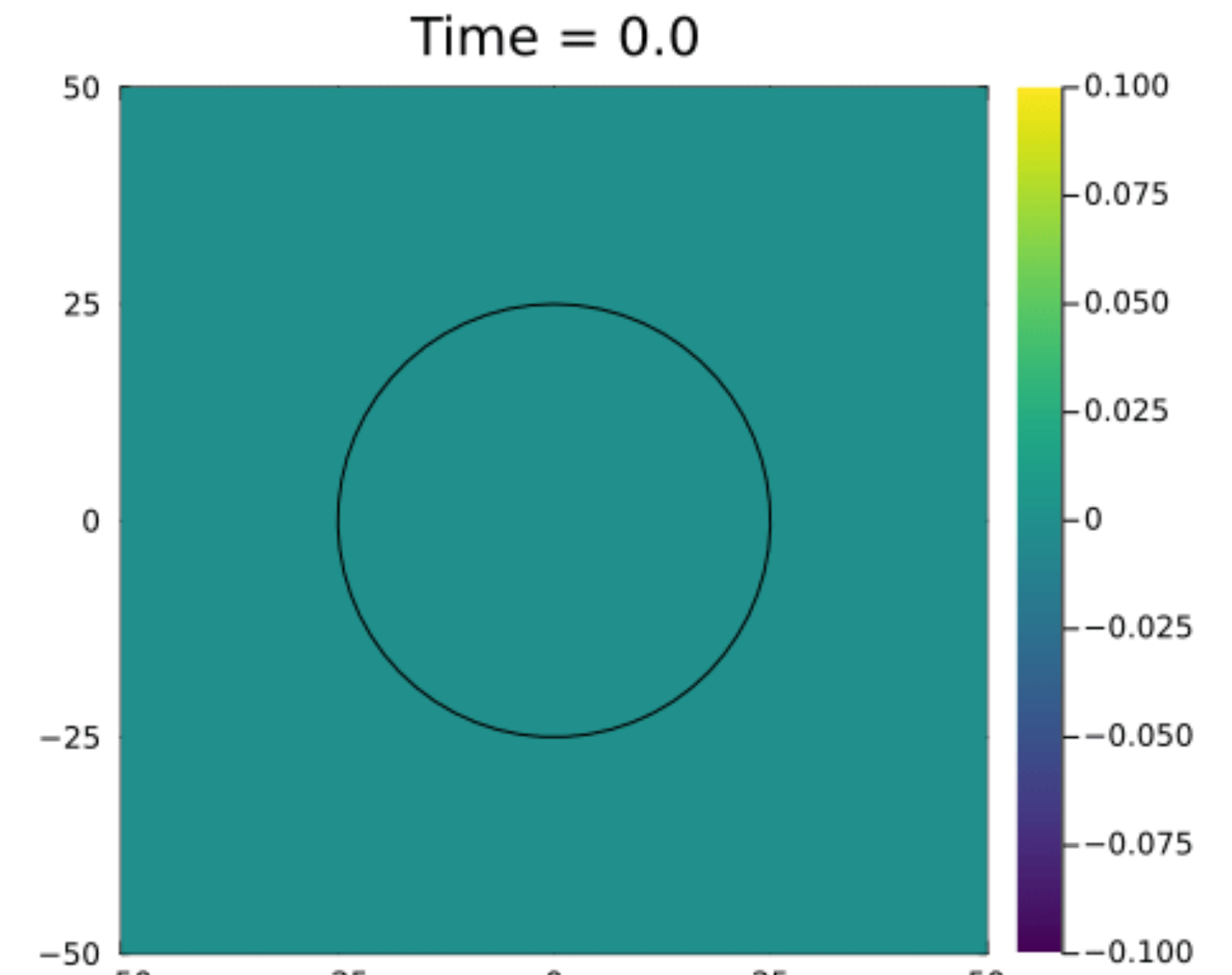




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Can we image Quantum Black Holes?

the case of q-metric

$$ds^2 = - \left(1 - \frac{2m}{r}\right)^{1+q} dt^2 + \left(1 - \frac{2m}{r}\right)^{-1-q} \left(\frac{r^2 - 2mr + m^2 \sin^2 \theta}{r^2 - 2mr}\right)^{-q(2+q)} dr^2 \\ + \left(1 - \frac{2m}{r}\right)^{-q} r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

- Generalized to spinning spacetime by Toktarbay & Quevedo 2014
- Modified the mass and spin multipoles of the Kerr spacetime

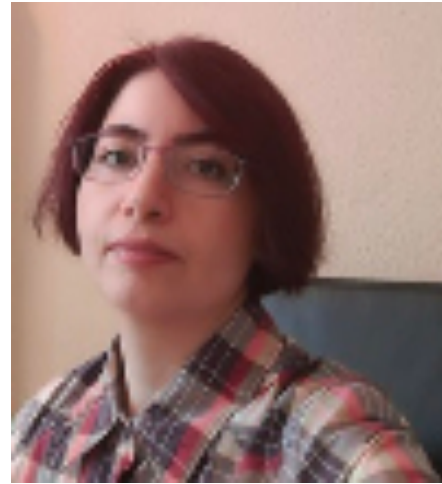
$$M_0 = m + q\sigma,$$

$$M_2 = -m^3 - 3m^2 q\sigma - m(q^2 - 1)\sigma^2 - \frac{1}{3}q(q^2 - 7)\sigma^3,$$

$$J_1 = a(m + 2q\sigma),$$

$$J_3 = -a[3m^3 + 12m^2 q\sigma + 3m(3q^2 - 1)\sigma^2 + 2q(q^2 - 4)\sigma^3] / 3,$$

$$\sigma = \sqrt{m^2 - a^2}$$



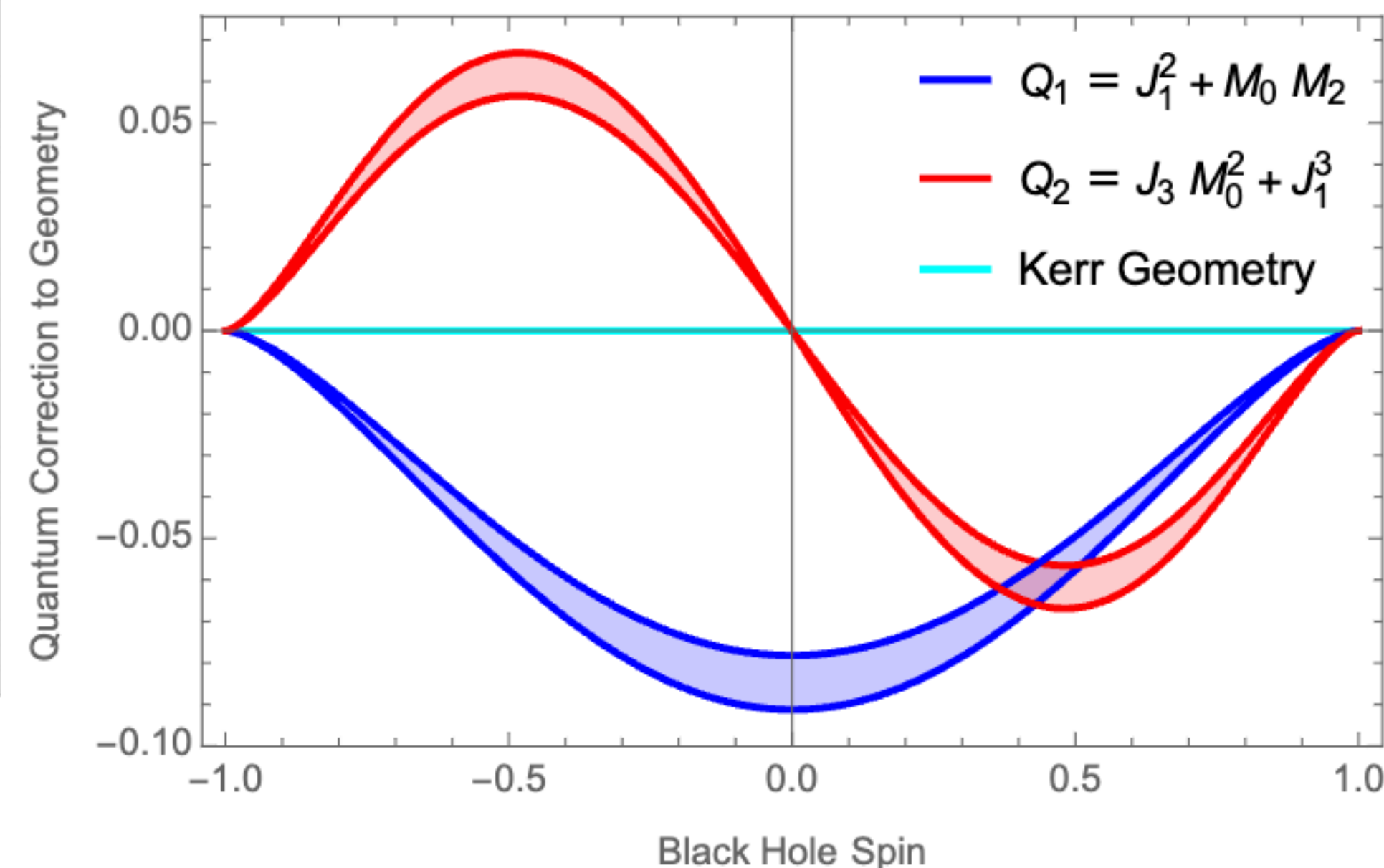
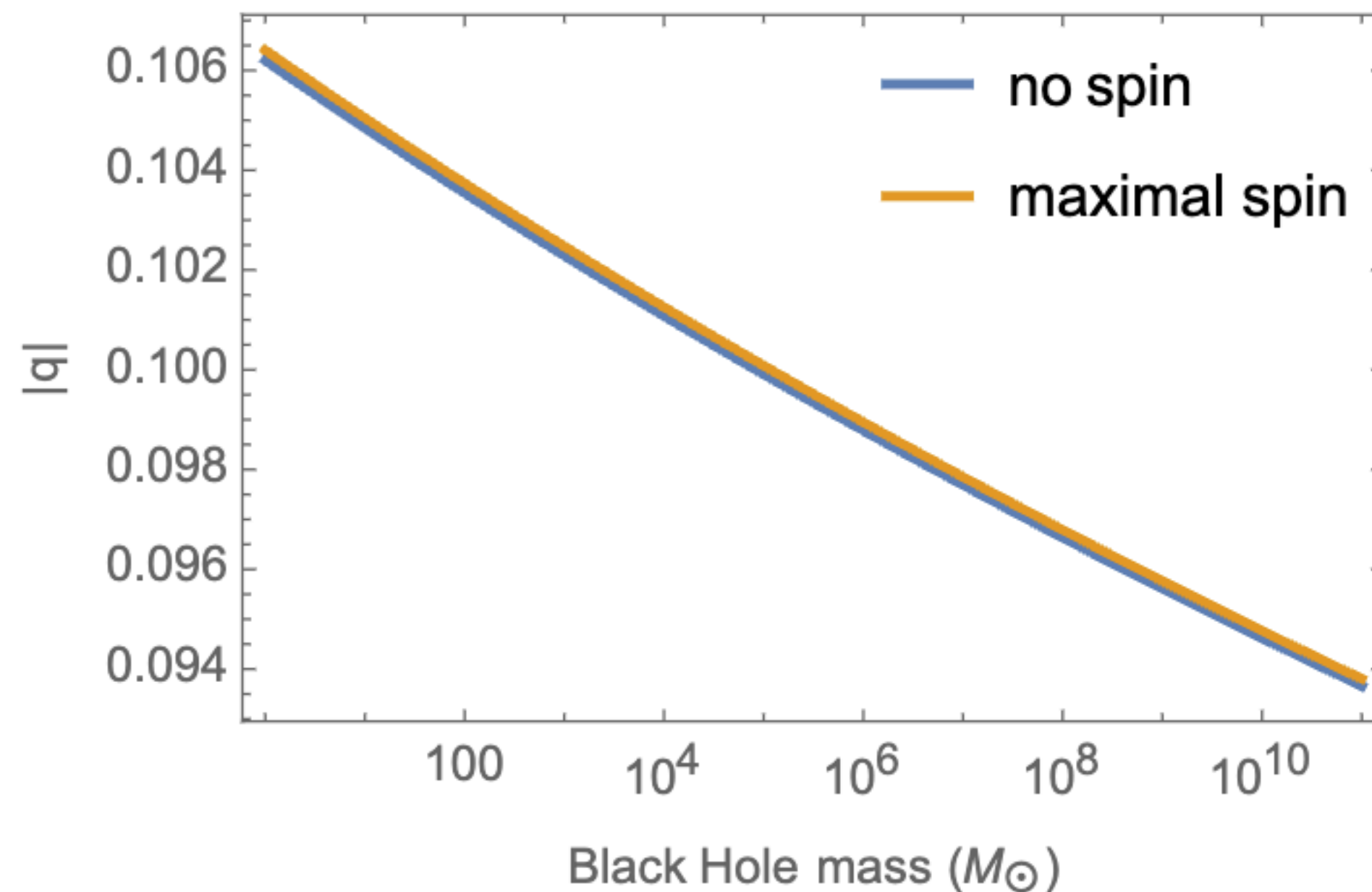
Imaging Quantum Black Holes?

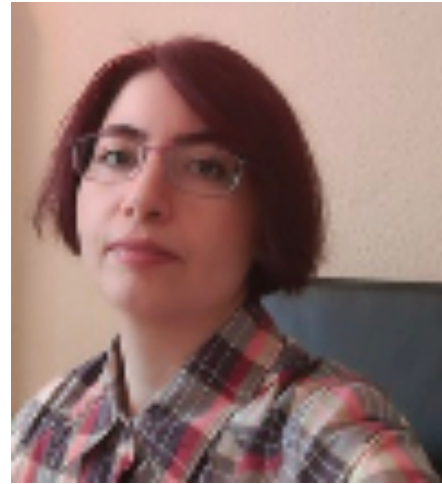
Faraji, Oshita, NA, et al. (in prep.)

- Quantum effects violate the no-hair theorem
 $\Rightarrow$ mass and spin multipoles differ from Kerr

- Example: q-metric

$$t_{\text{echo}} \lesssim \frac{\ln S}{2\pi T_H} \Rightarrow q^2 \gtrsim \frac{2}{\ln S}.$$





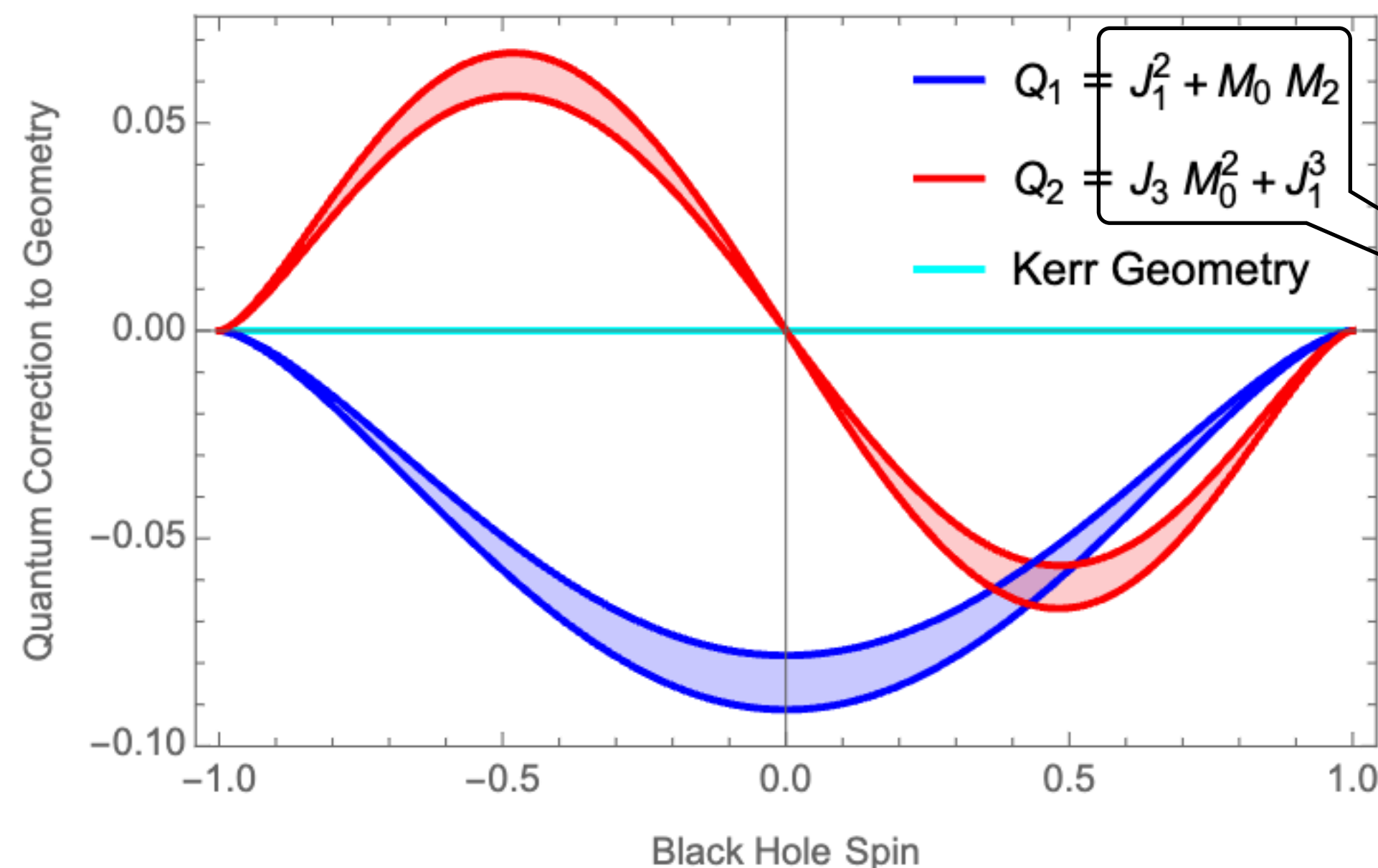
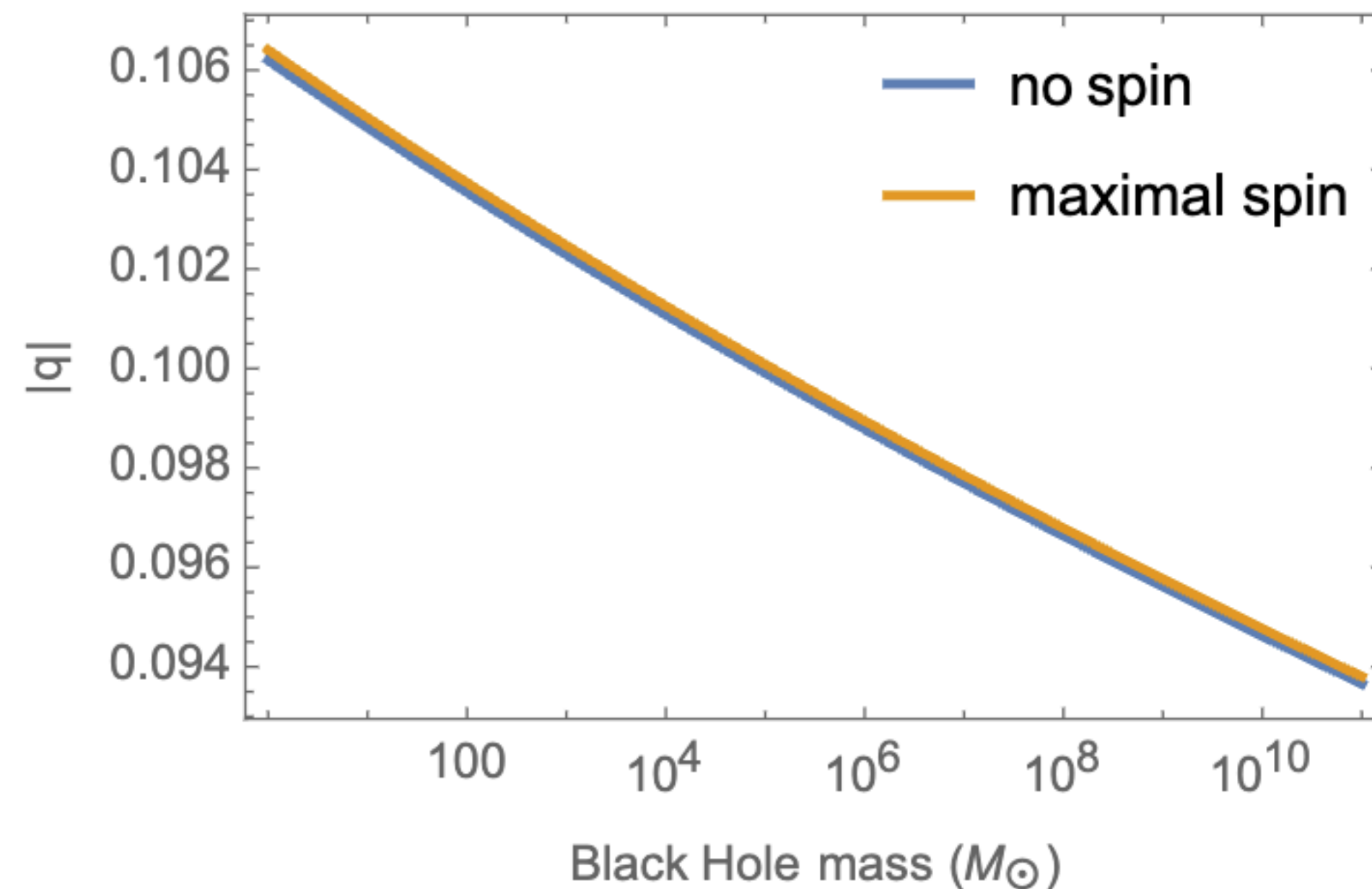
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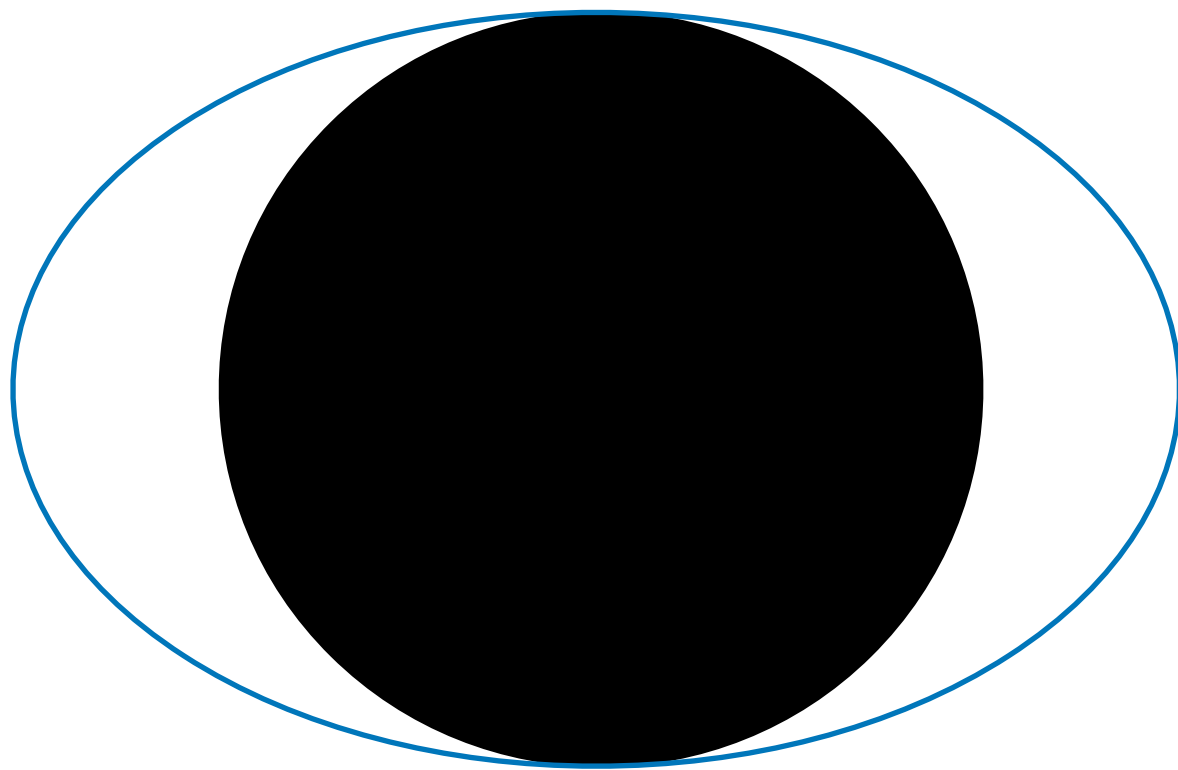
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mass and spin moments

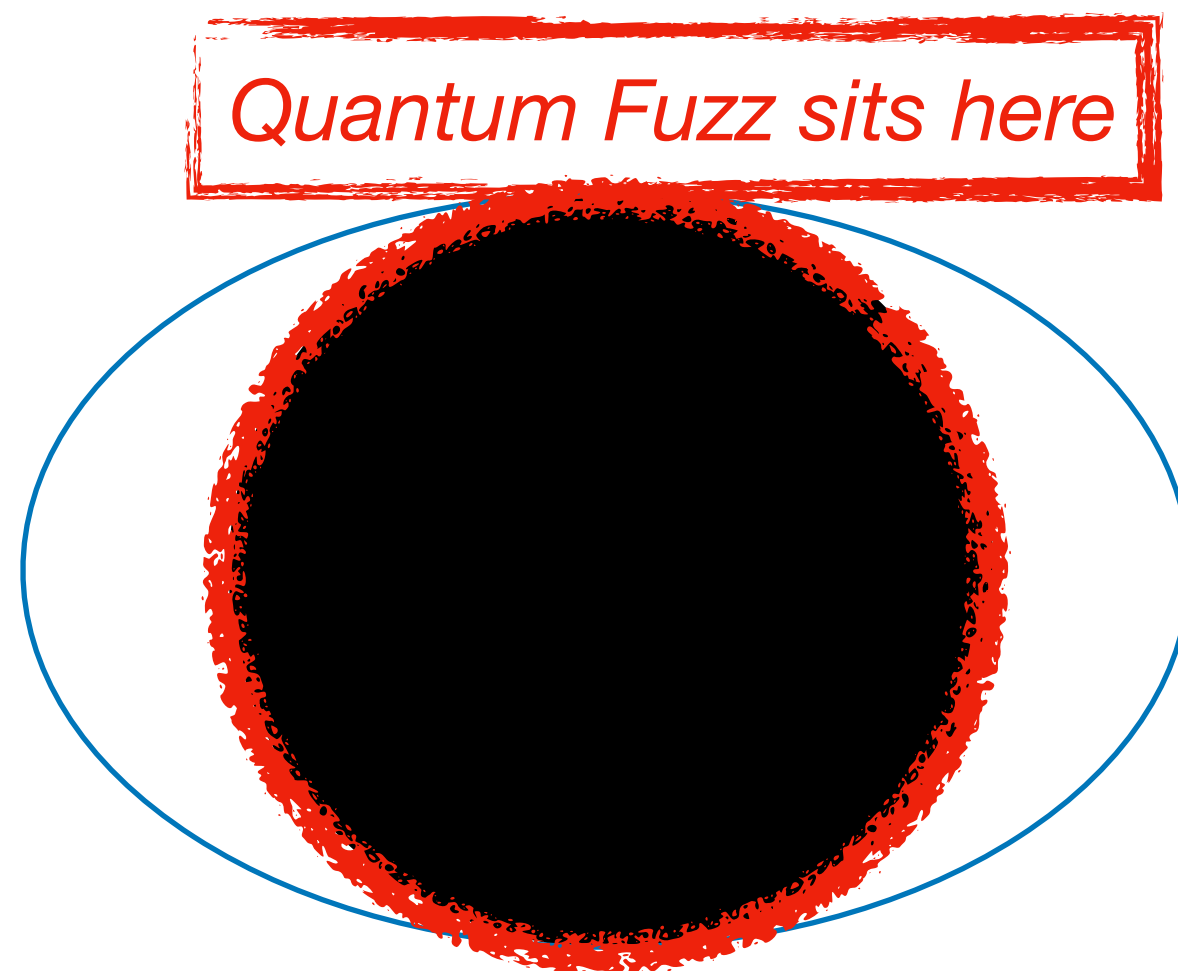
Upshot:

Quantum Violation of no-hair theorem



Upshot:

Quantum Violation of no-hair theorem

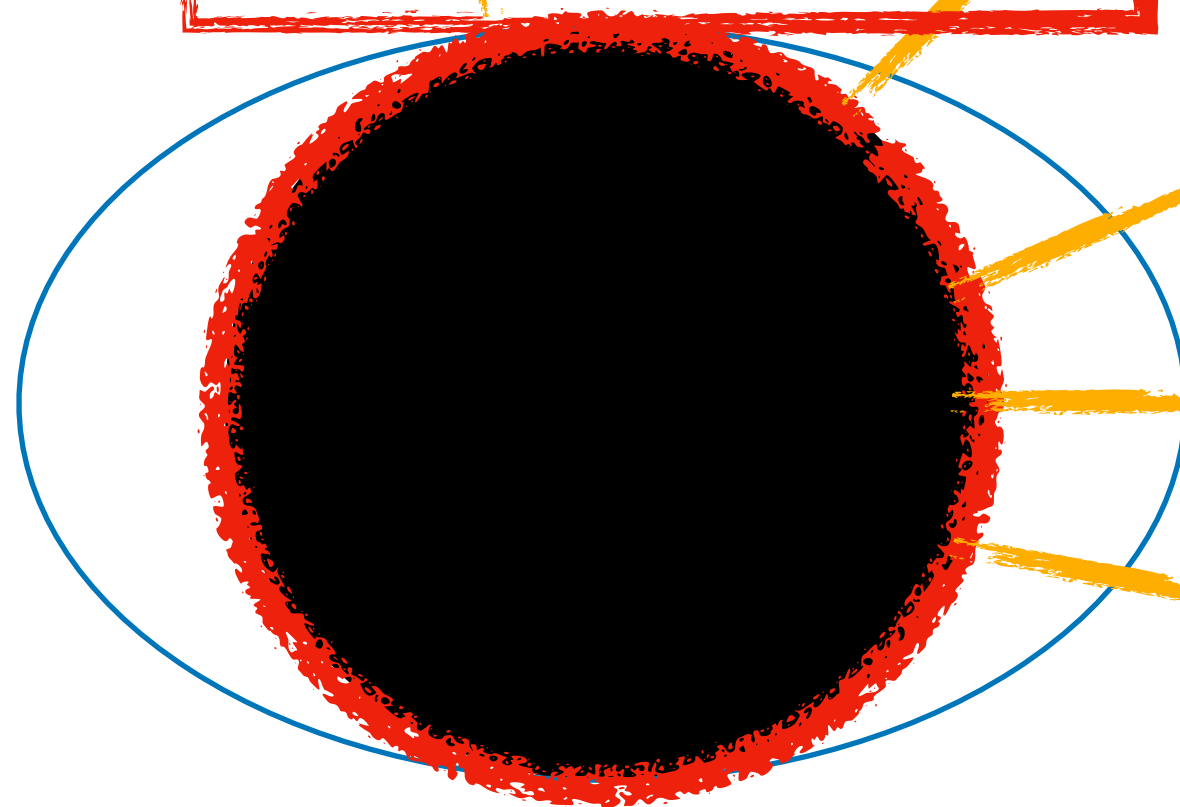


Upshot:

Quantum Violation of no-hair theorem

But it causes deviation from Kerr multipoles here $\sim \frac{1}{\sqrt{\log(\text{entropy})}}$

Quantum Fuzz sits here



Upshot:

Quantum Violation of no-hair theorem

But it causes deviation from Kerr multipoles here

$$\sim \frac{1}{\sqrt{\log(\text{entropy})}}$$

Also see:

Quantum Fuzz sits here

arXiv > gr-qc > arXiv:2309.11491

General Relativity and Quantum Cosmology

[Submitted on 20 Sep 2023 (v1), last revised 21 Jun 2024 (this version, v3)]

Vacuum Spacetime With Multipole Moments: The Minimal Size Conjecture, Black Hole Shadow, and Gravitational Wave Observables

Shammi Tahura, Hassan Khalvati, Huan Yang

arXiv > hep-th > arXiv:2103.12075

High Energy Physics – Theory

[Submitted on 22 Mar 2021 (v1), last revised 20 Oct 2021 (this version, v2)]

Fuzzball Shadows: Emergent Horizons from Microstructure

Fabio Bacchini, Daniel R. Mayerson, Bart Ripperda, Jordy Davelaar, Héctor Olivares, Thomas Hertog, Bert Verhocke

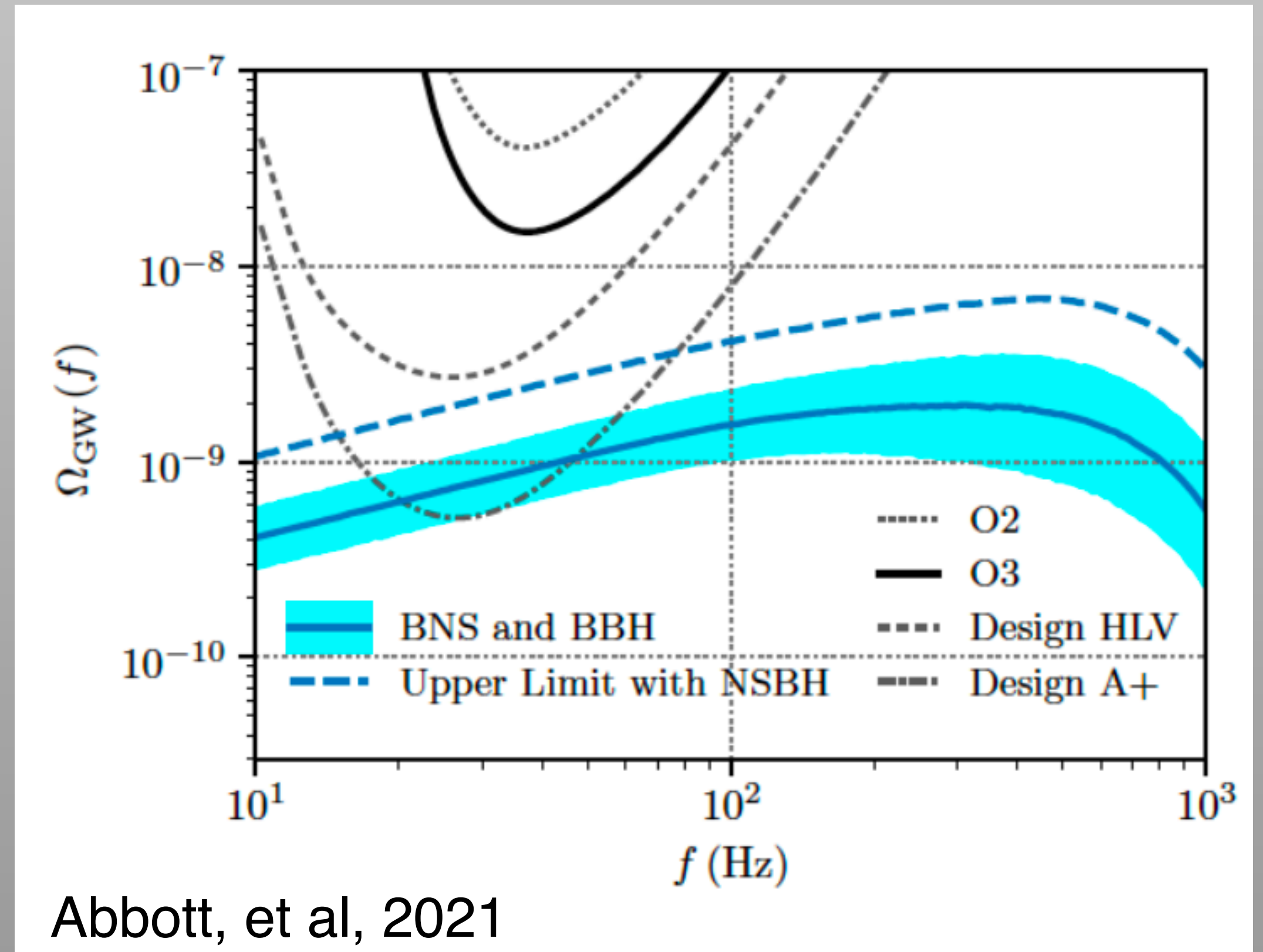
When we fail to see anything

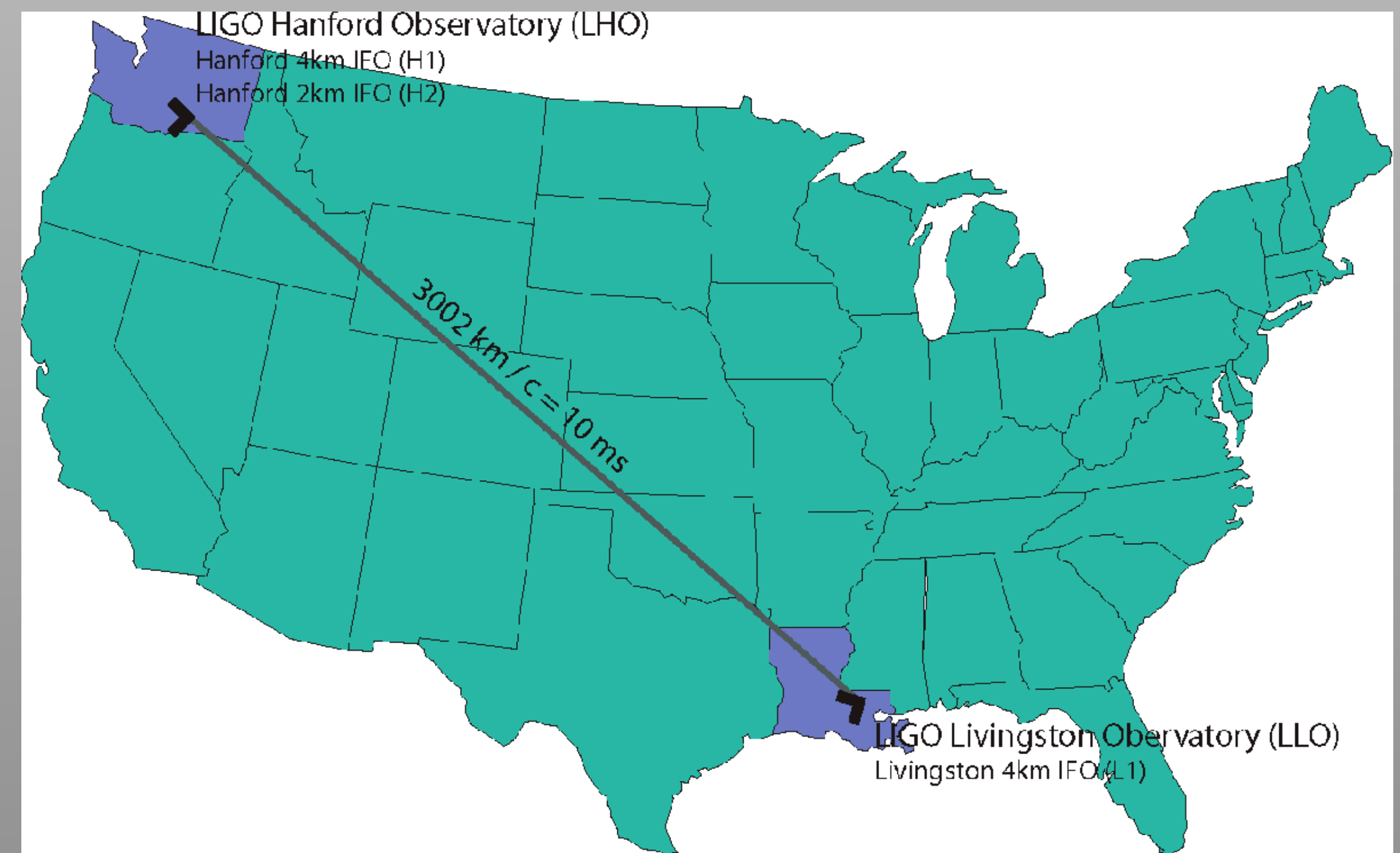
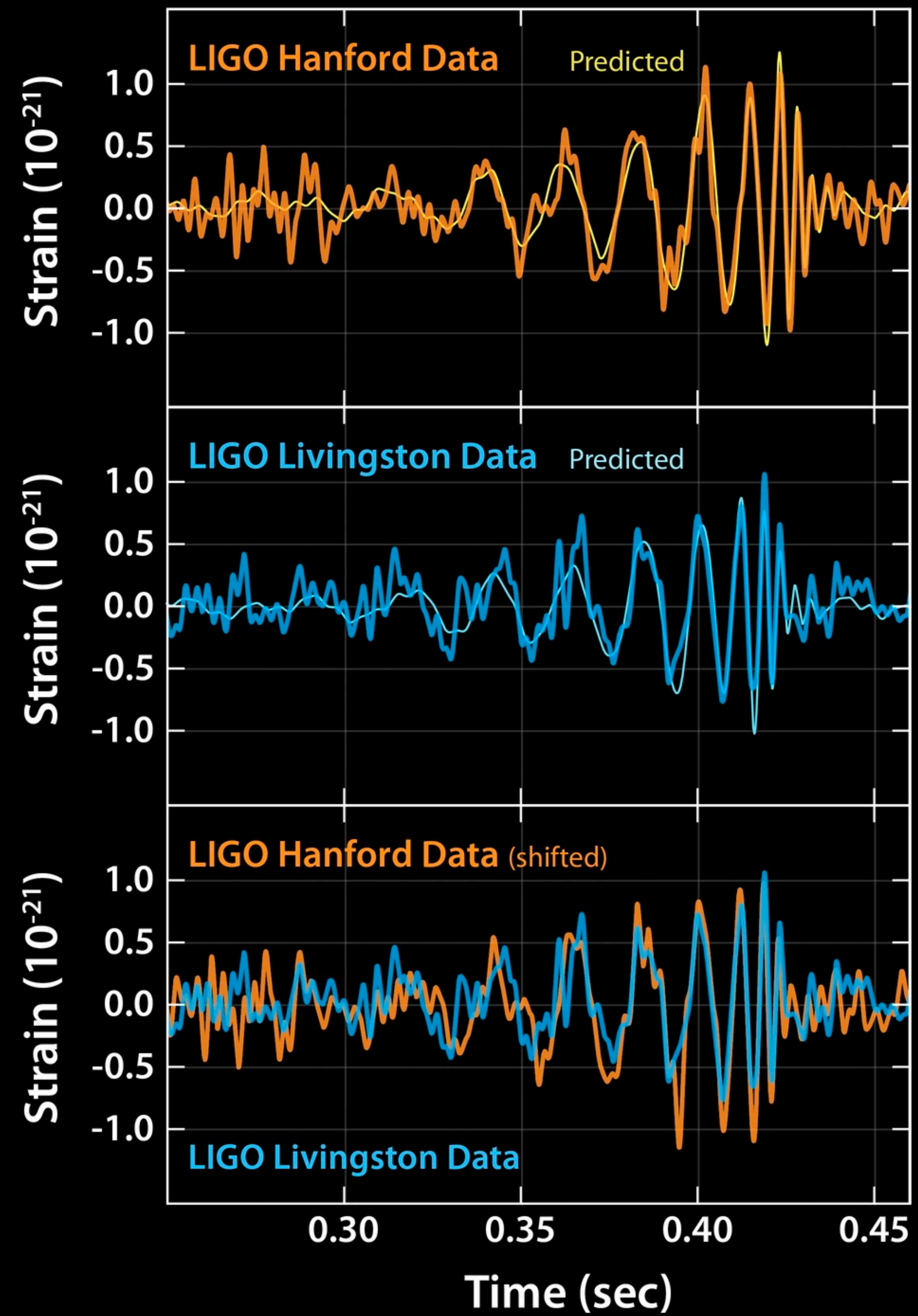
Subthreshold: what if we cannot detect the events?

Detecting the Stochastic Gravitational Wave background (SGWB) by

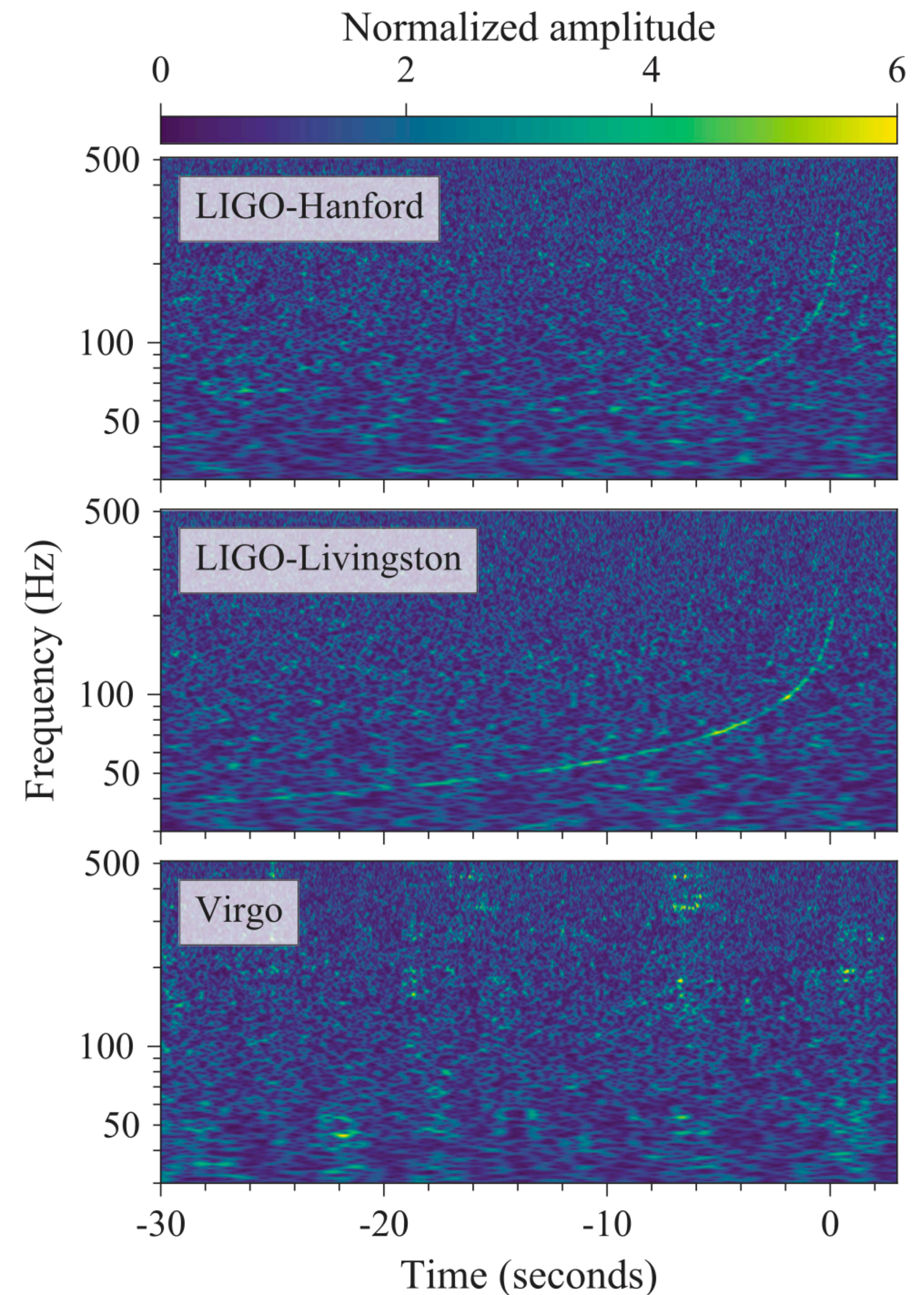
*detector signal **cross-power spectrum***

Time-delay between detectors (± 10 msec)
kills signal at frequency > 30 Hz (for
unknown sky position)





Note that spectrograms remain correlated on much longer times



Signal beyond power spectrum

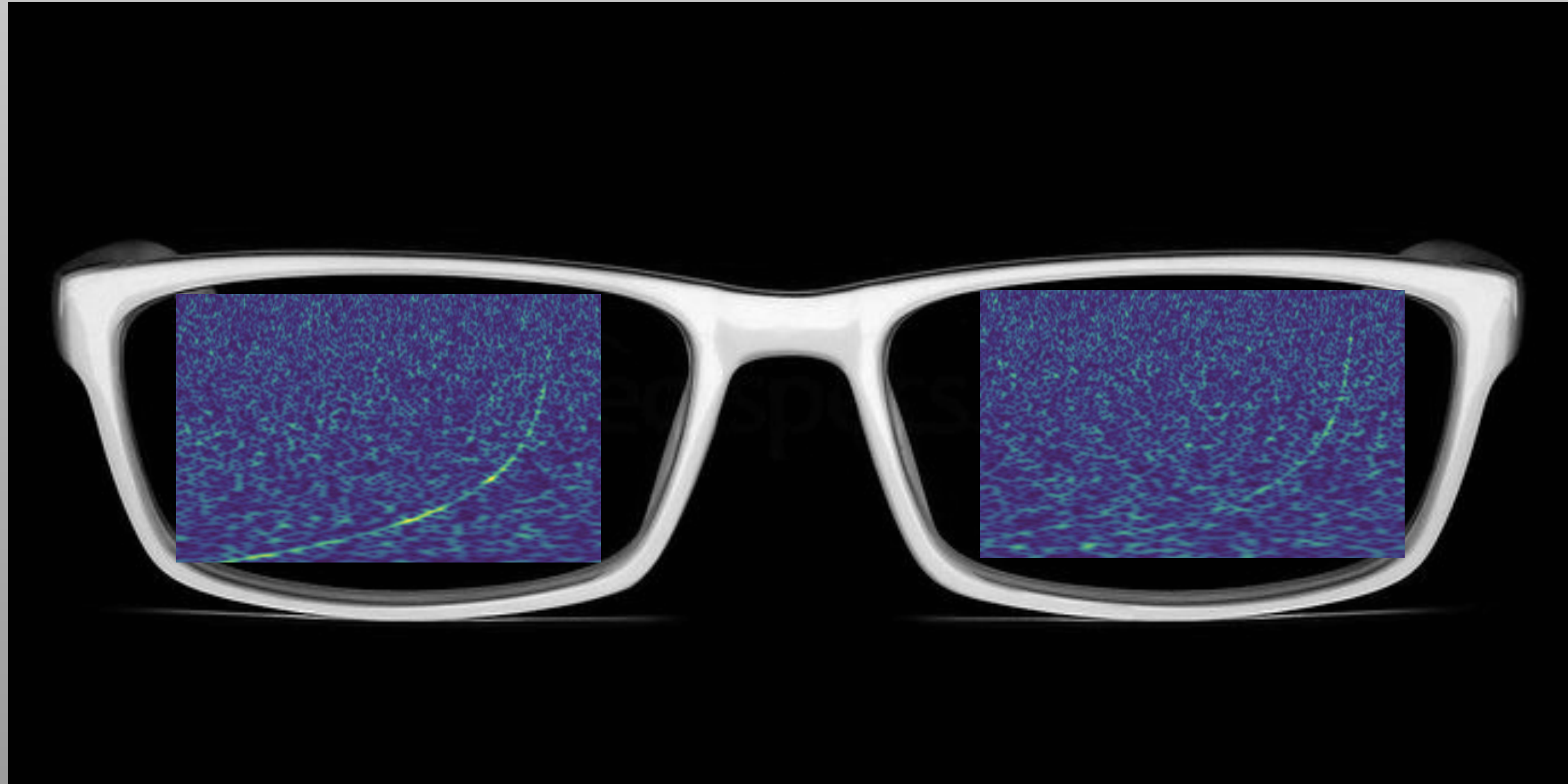
The temporal dependence of the signal is not captured *only* by the power spectrum

A new method:

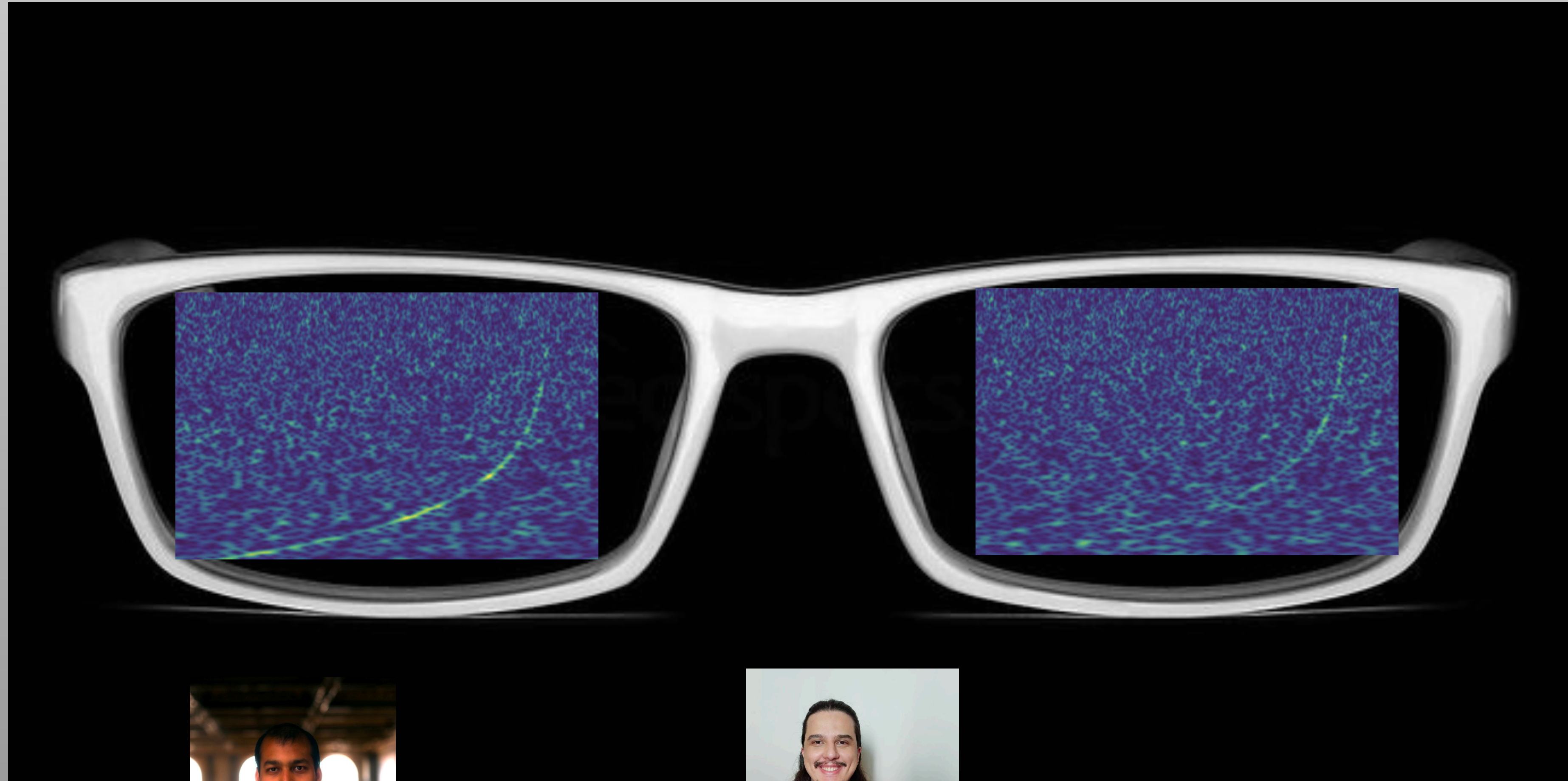
SpeCS: Spectrogram Correlated Stacking

Dey, Longo, Mukherjee & NA 2023

SpeCS: Spectrogram Correlated Stacking



SpeCS: Spectrogram Correlated Stacking



Ramit Dey (Western)



Luis Longo (FSU, Jena)

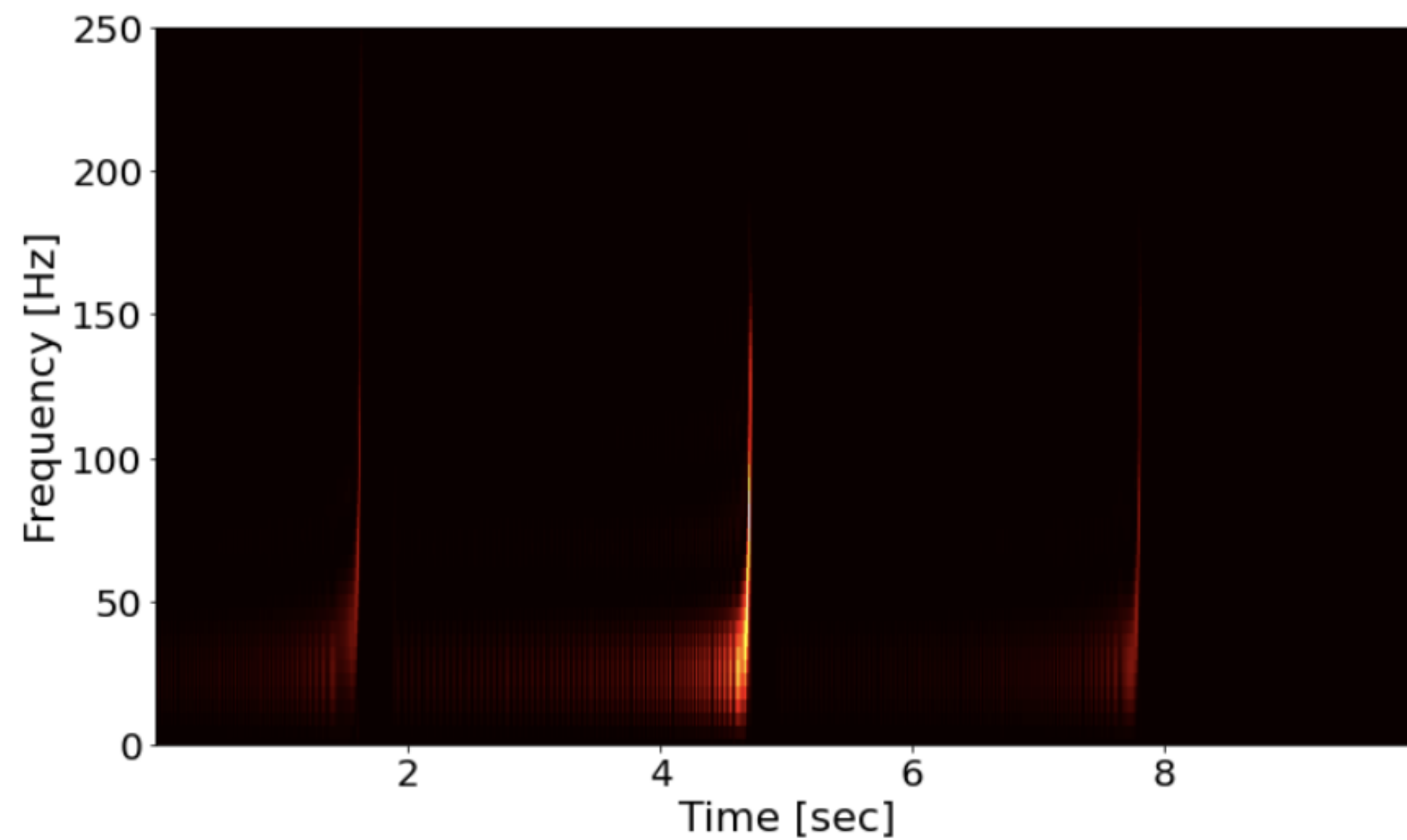


Suvodip Mukherjee (TIFR, India)

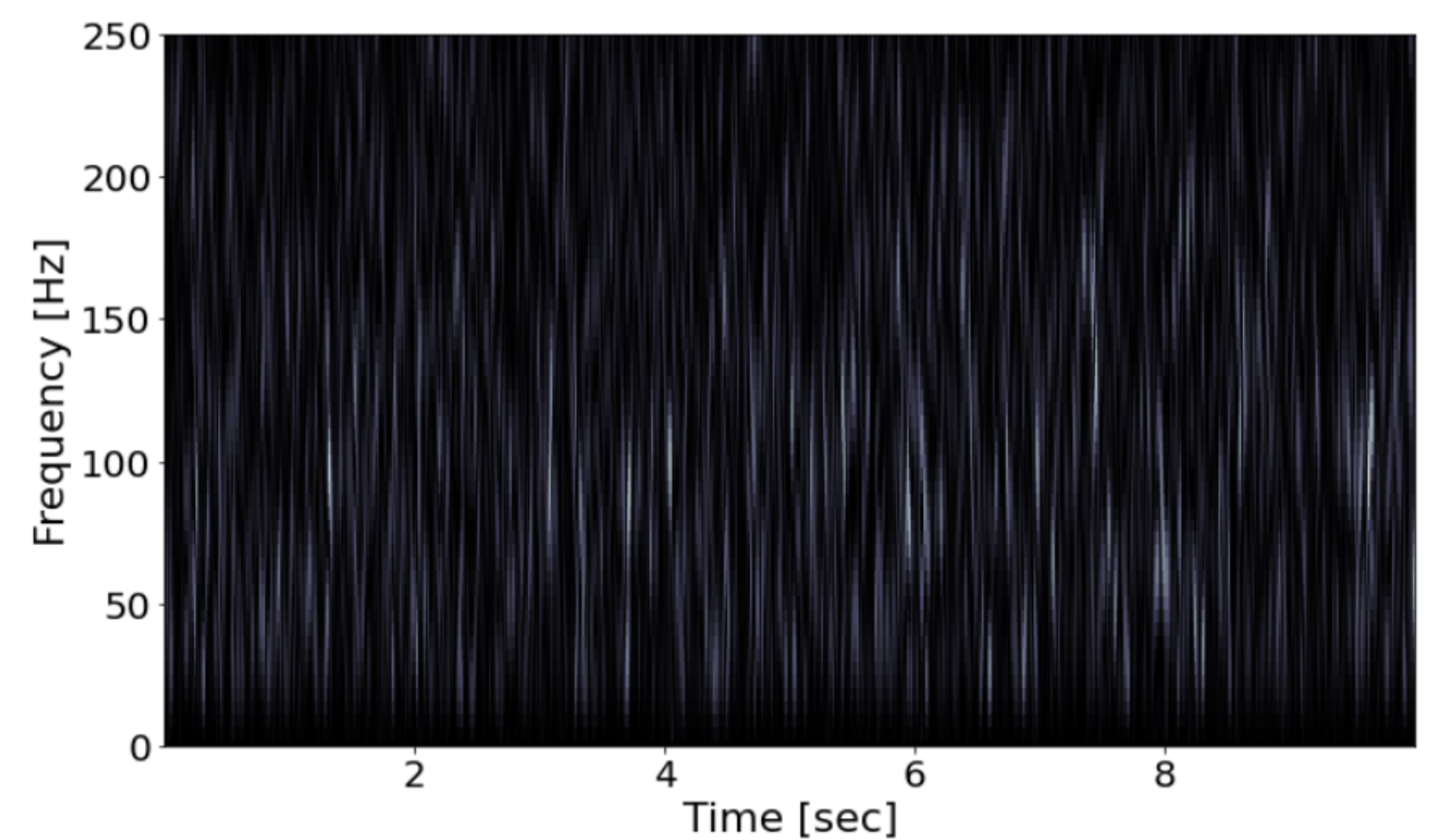


Implementation of SpeCS on injections

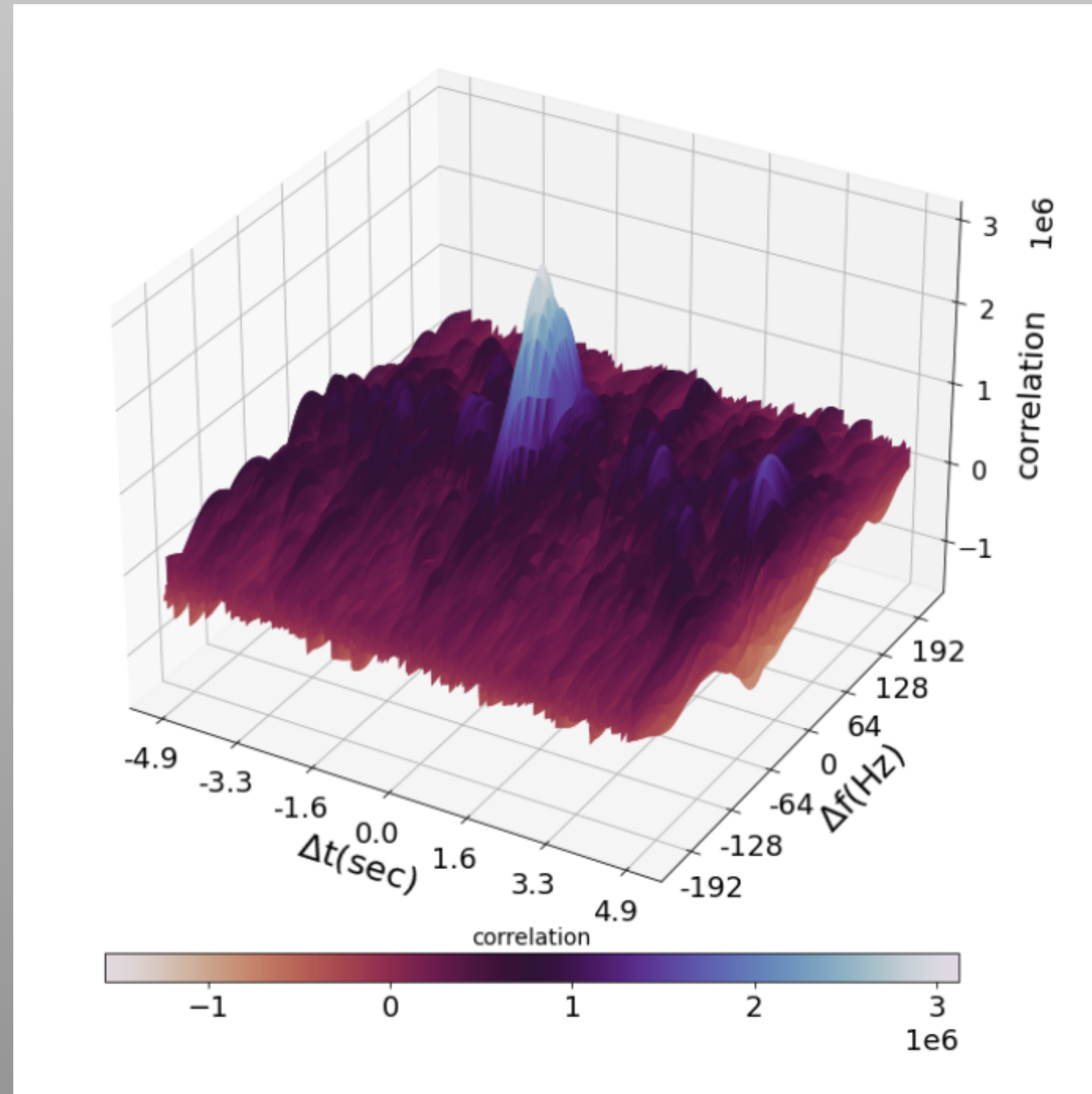
signal



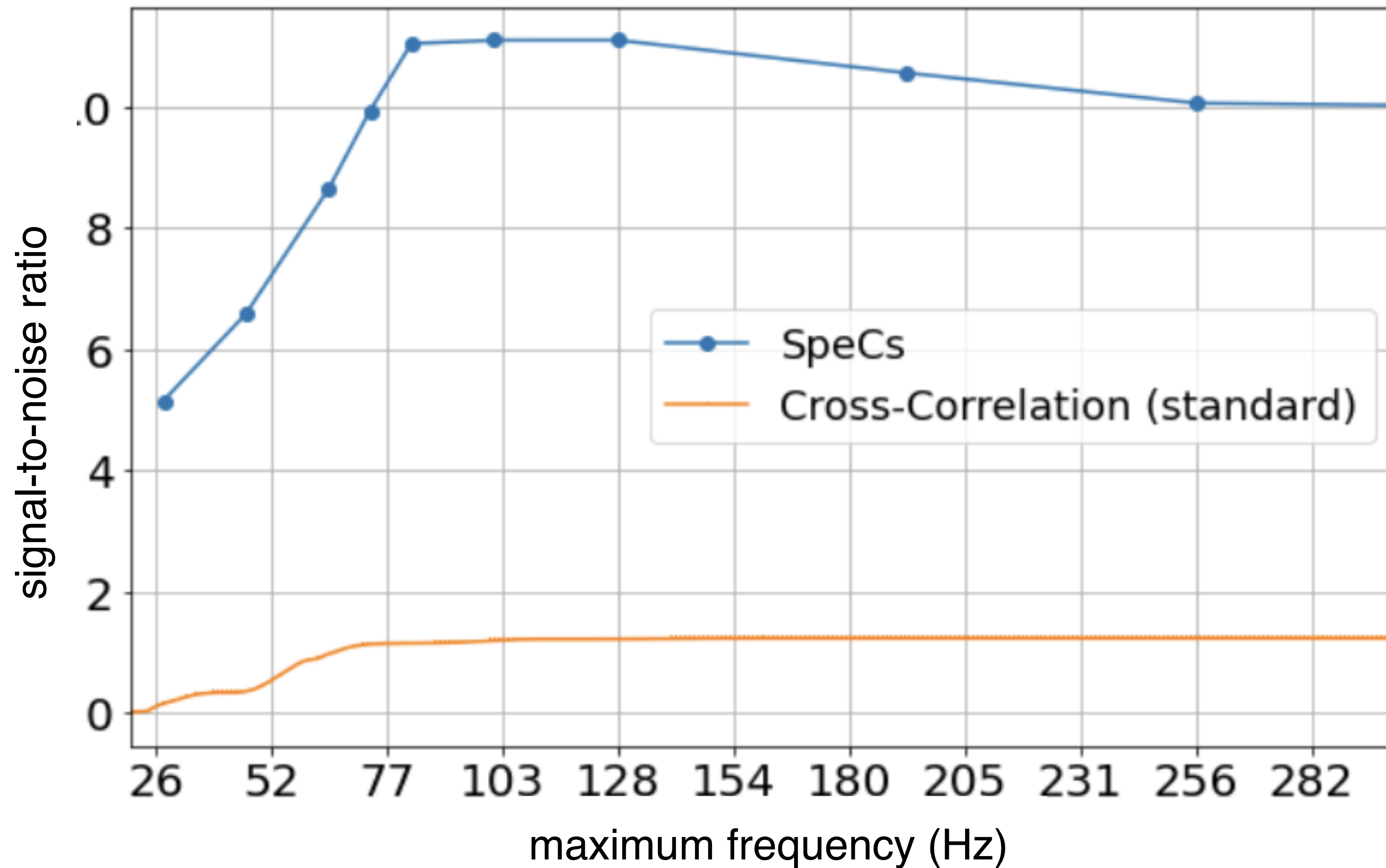
noise



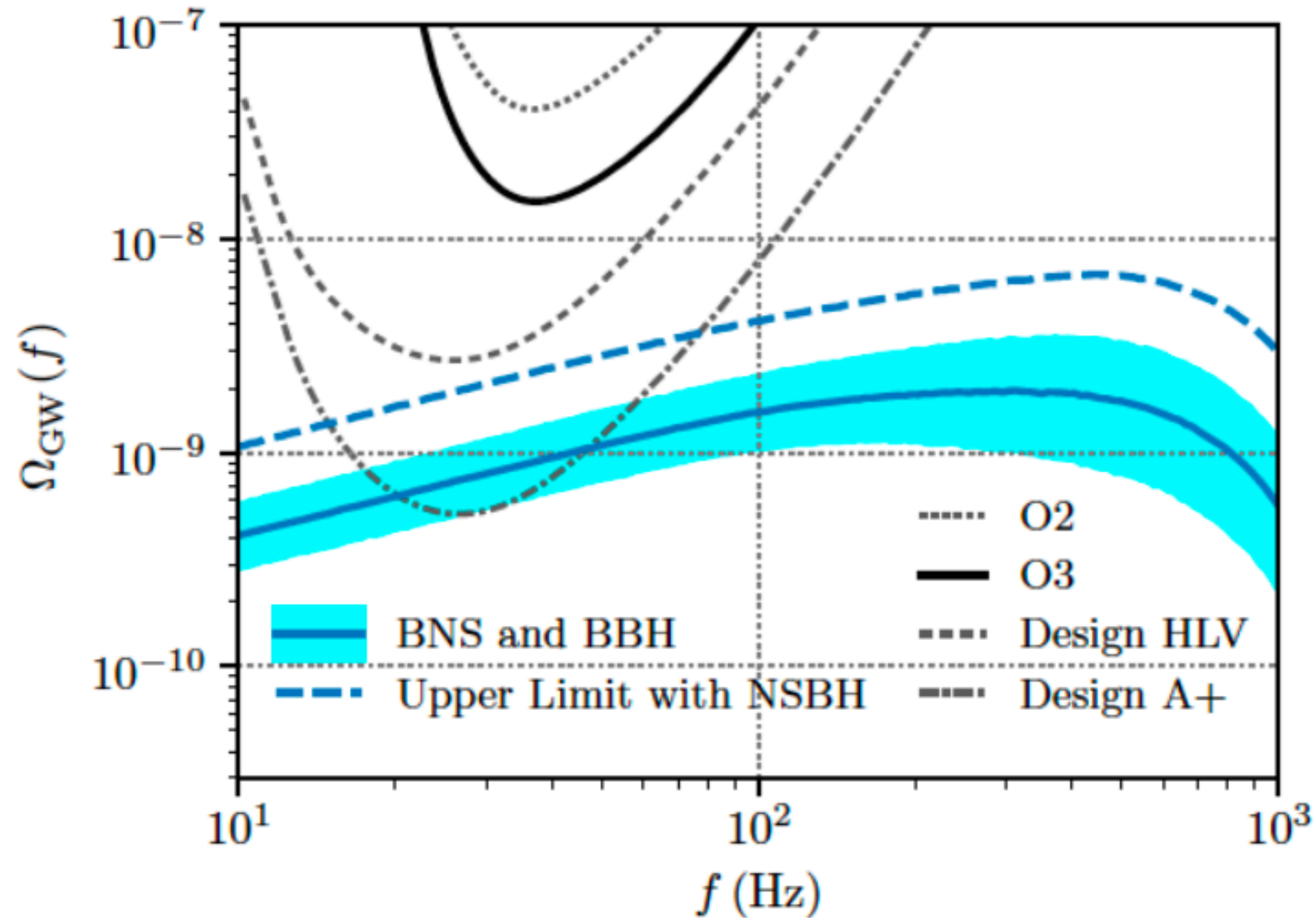
SpeCS: Results on the injections



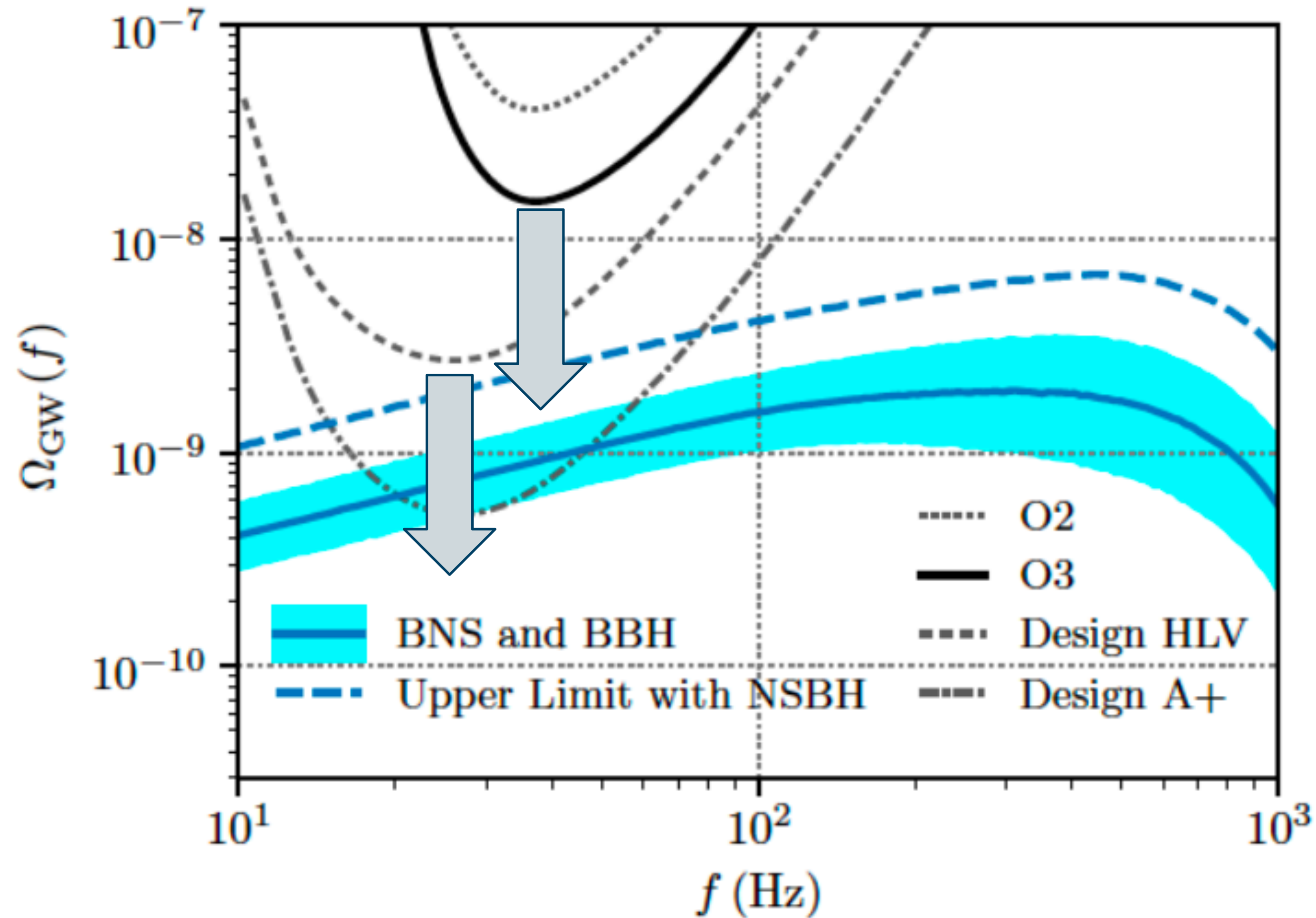
Comparison with the cross-correlation technique



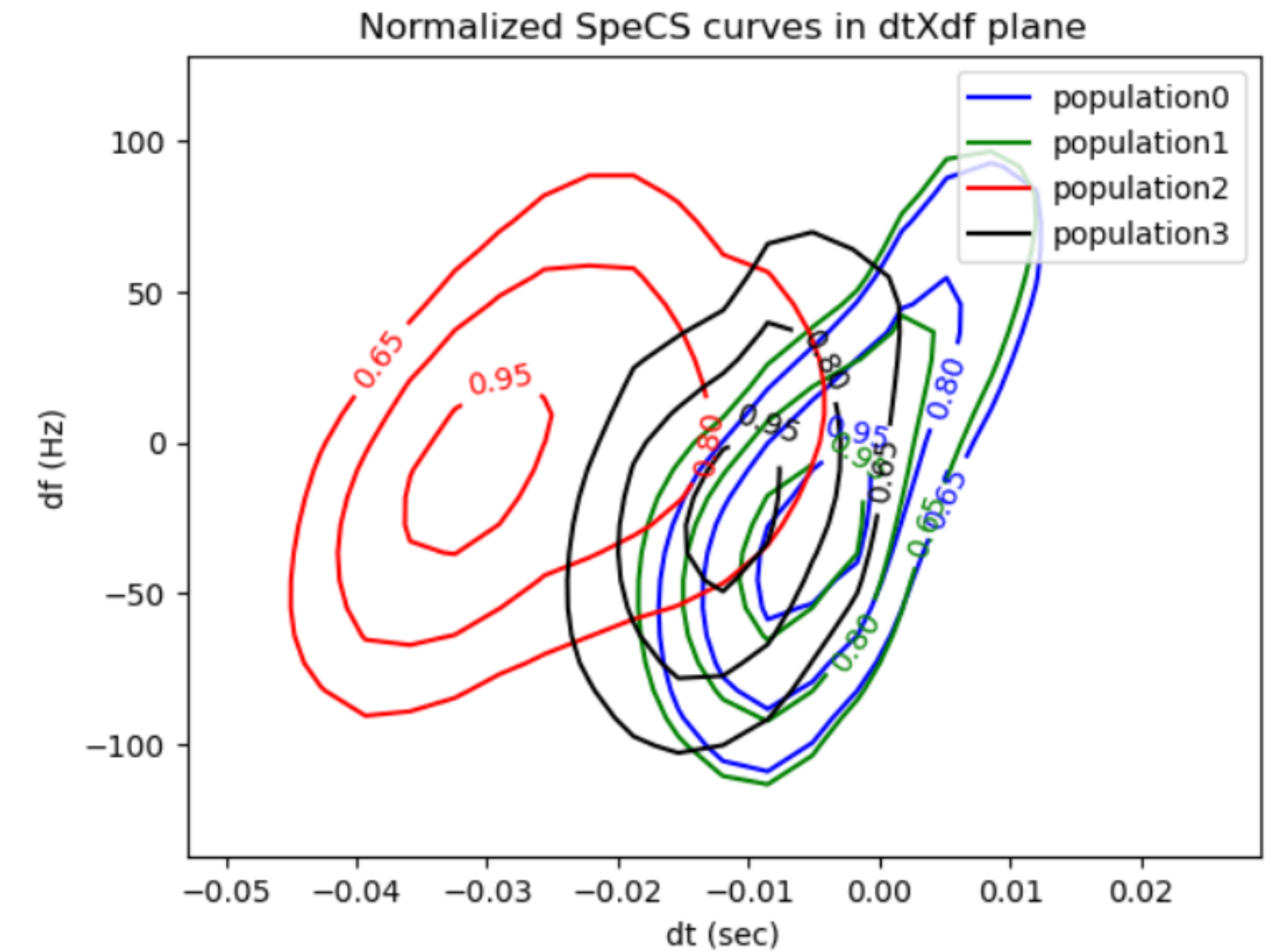
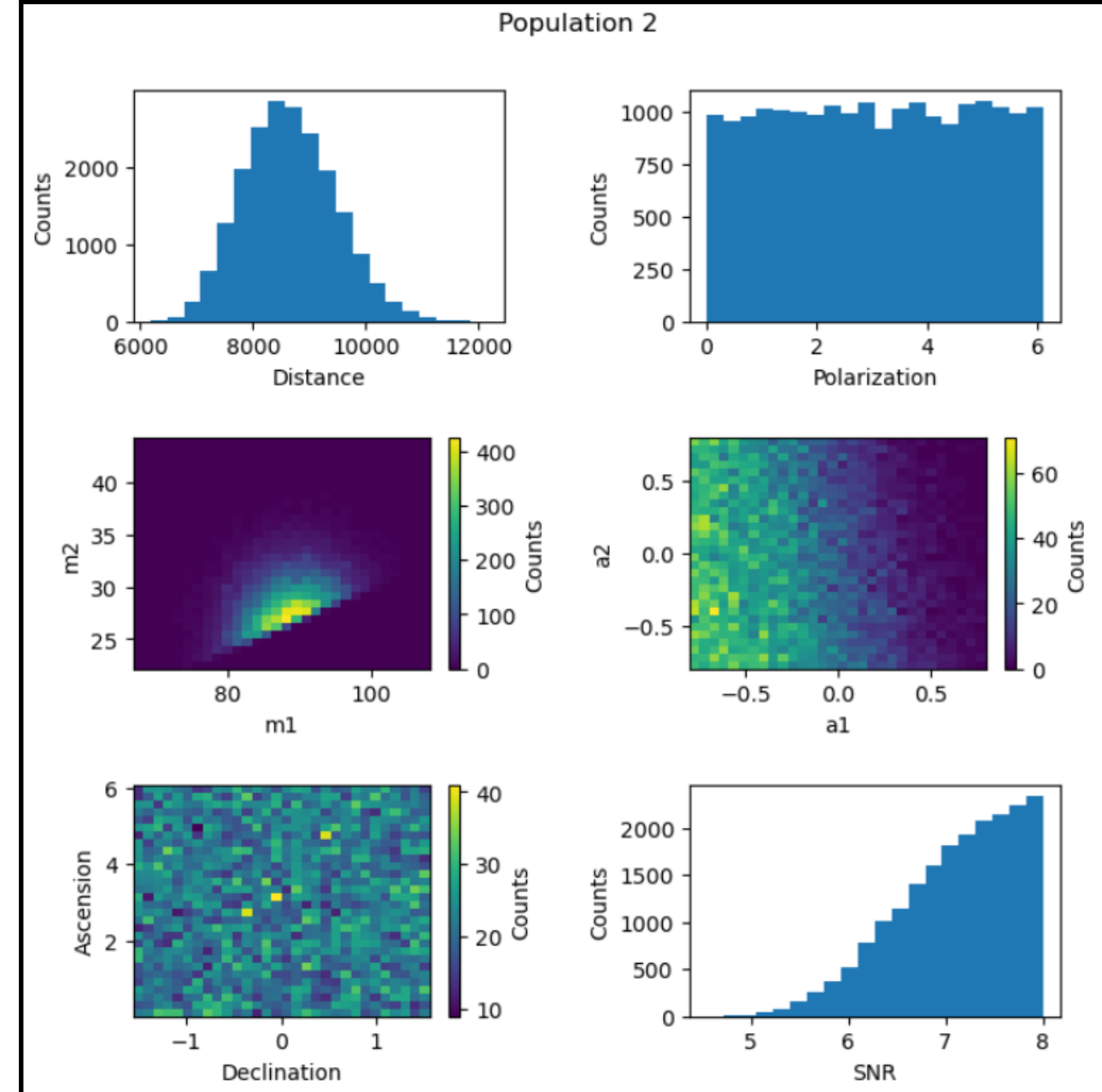
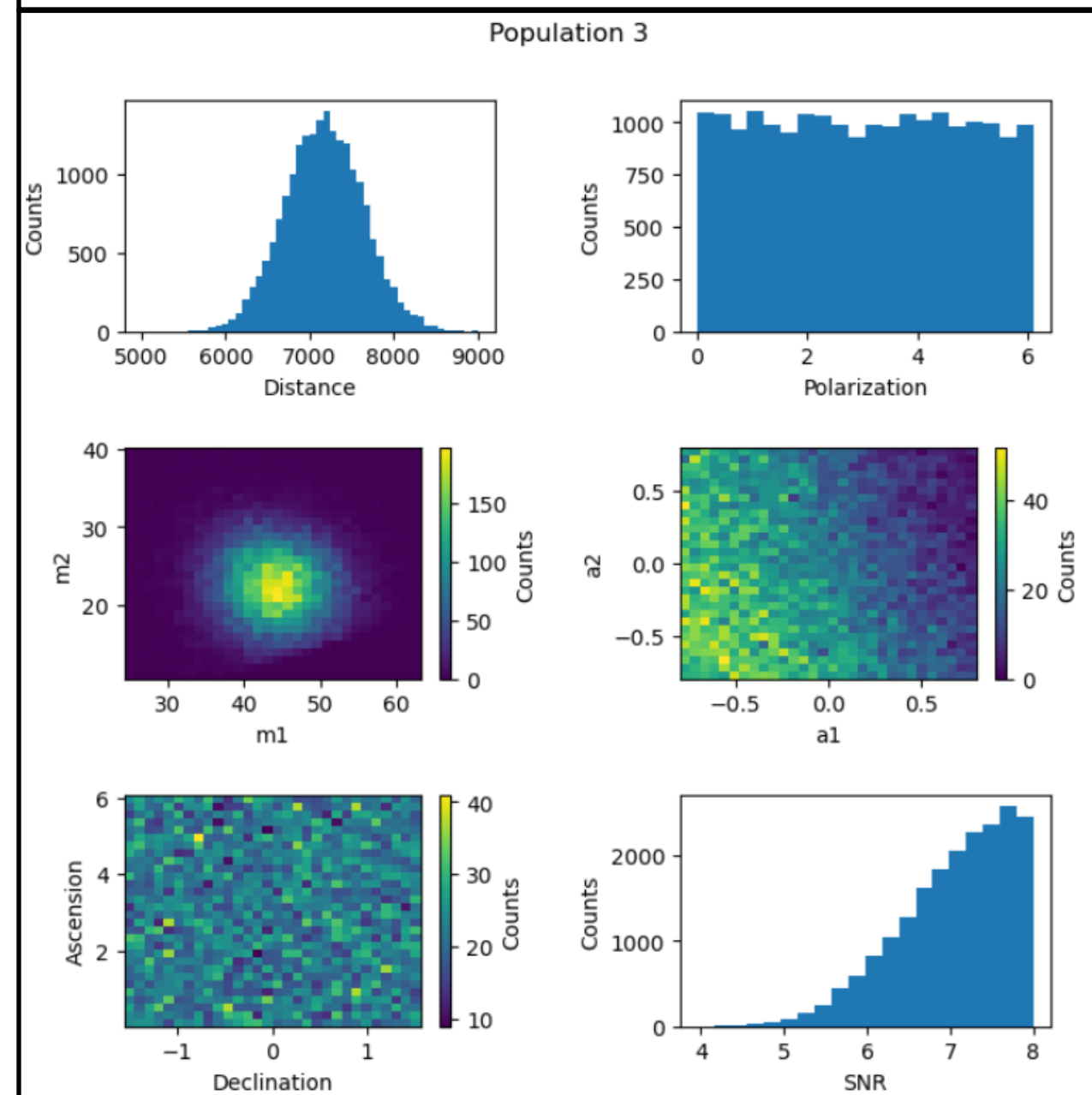
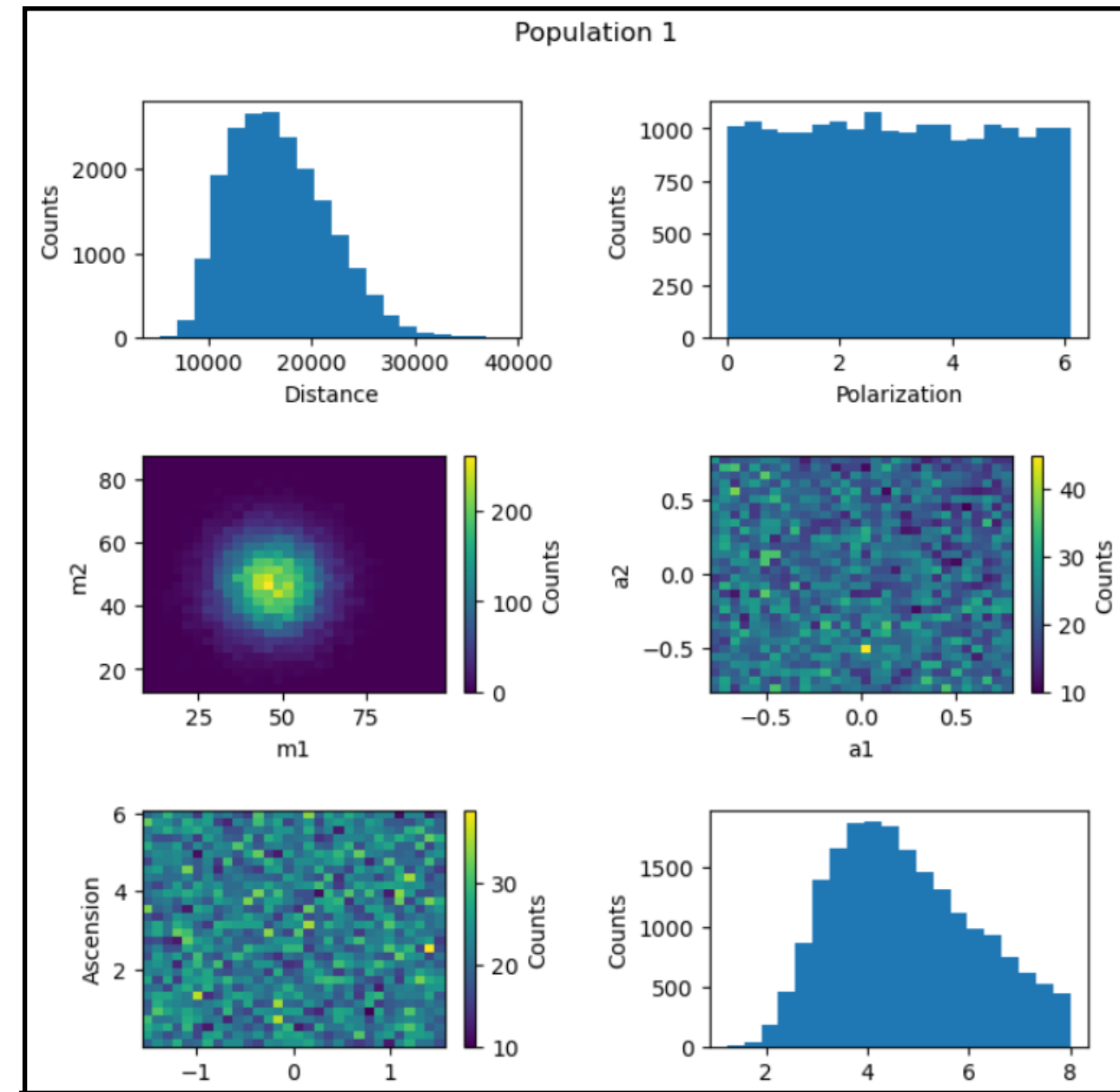
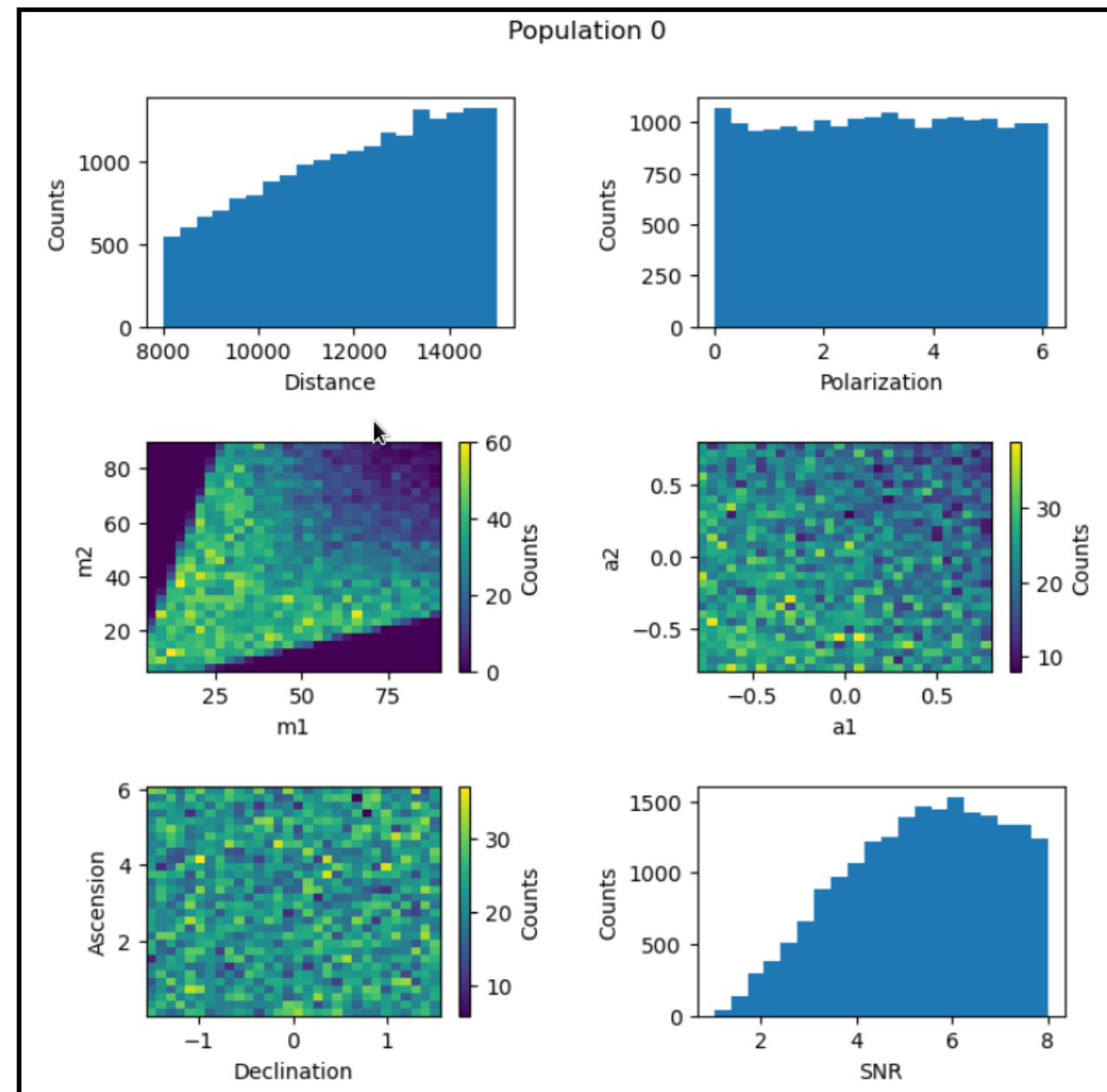
SpeCS improves (astrophysics) SNR by x8



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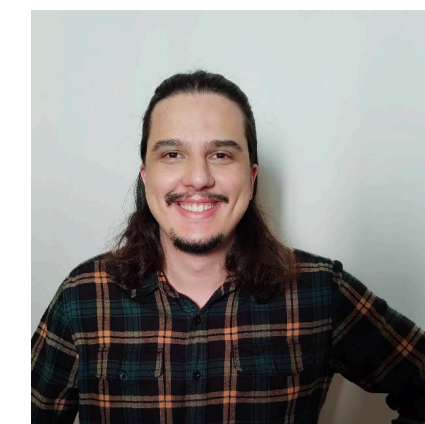
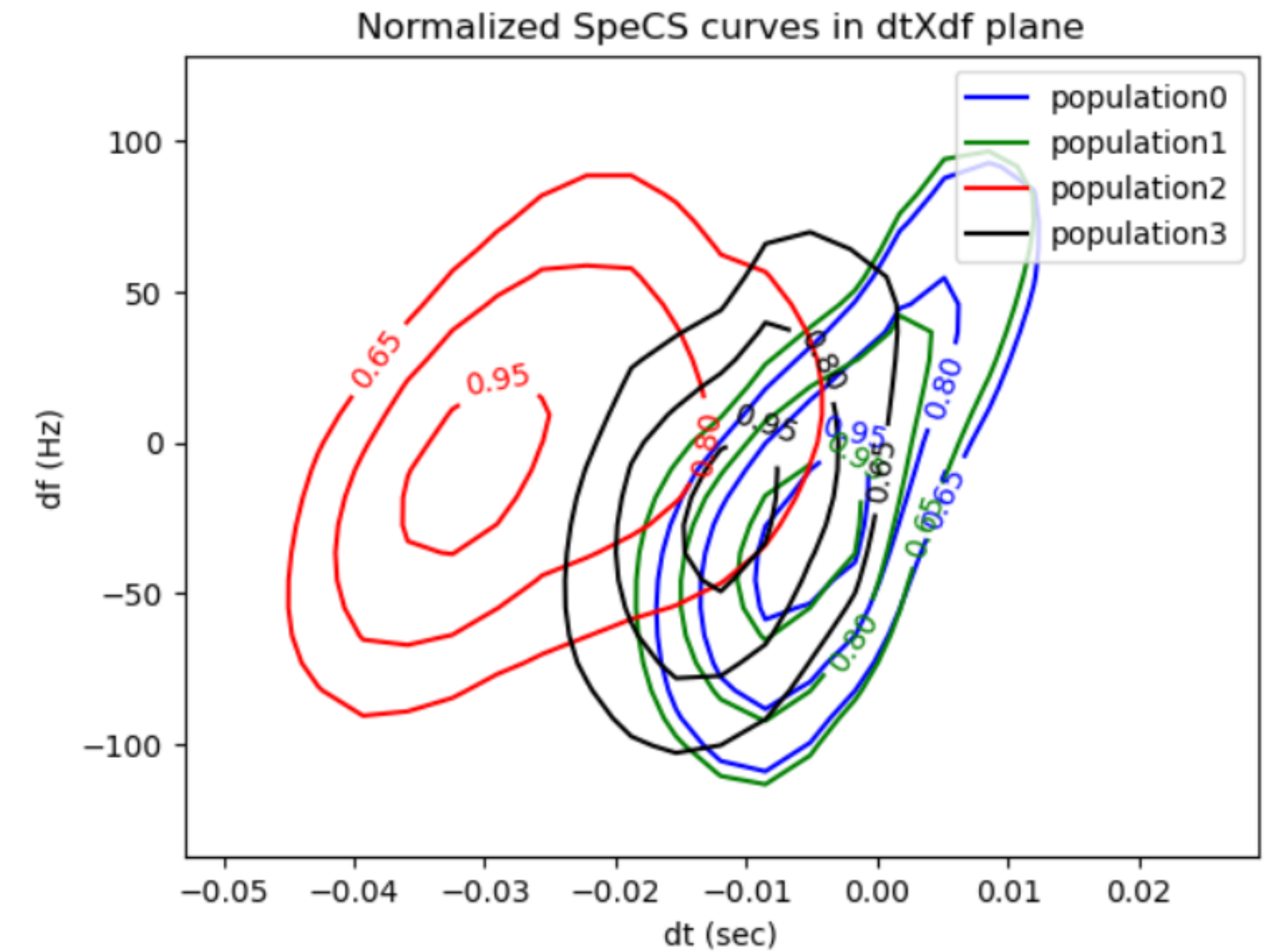
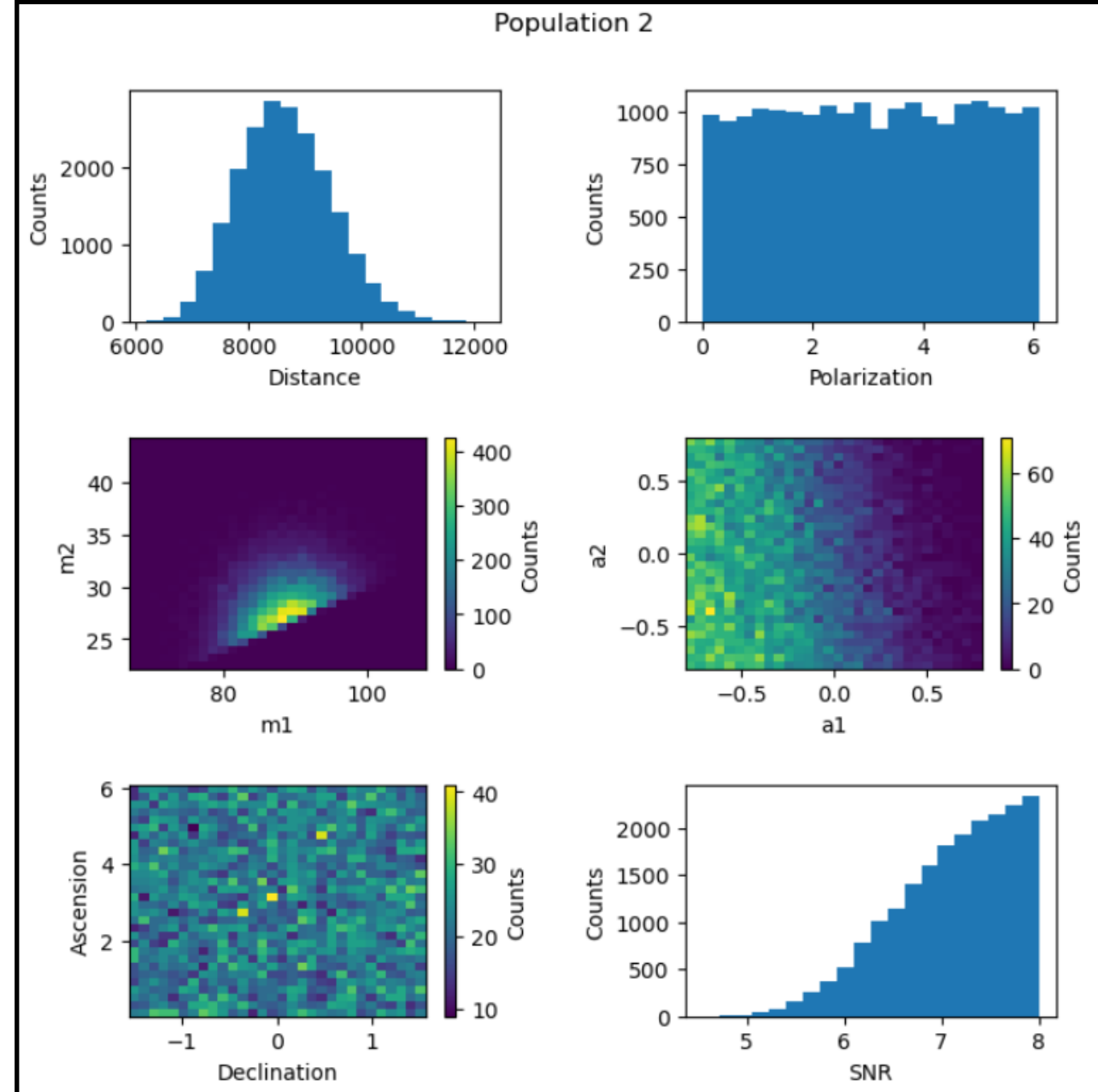
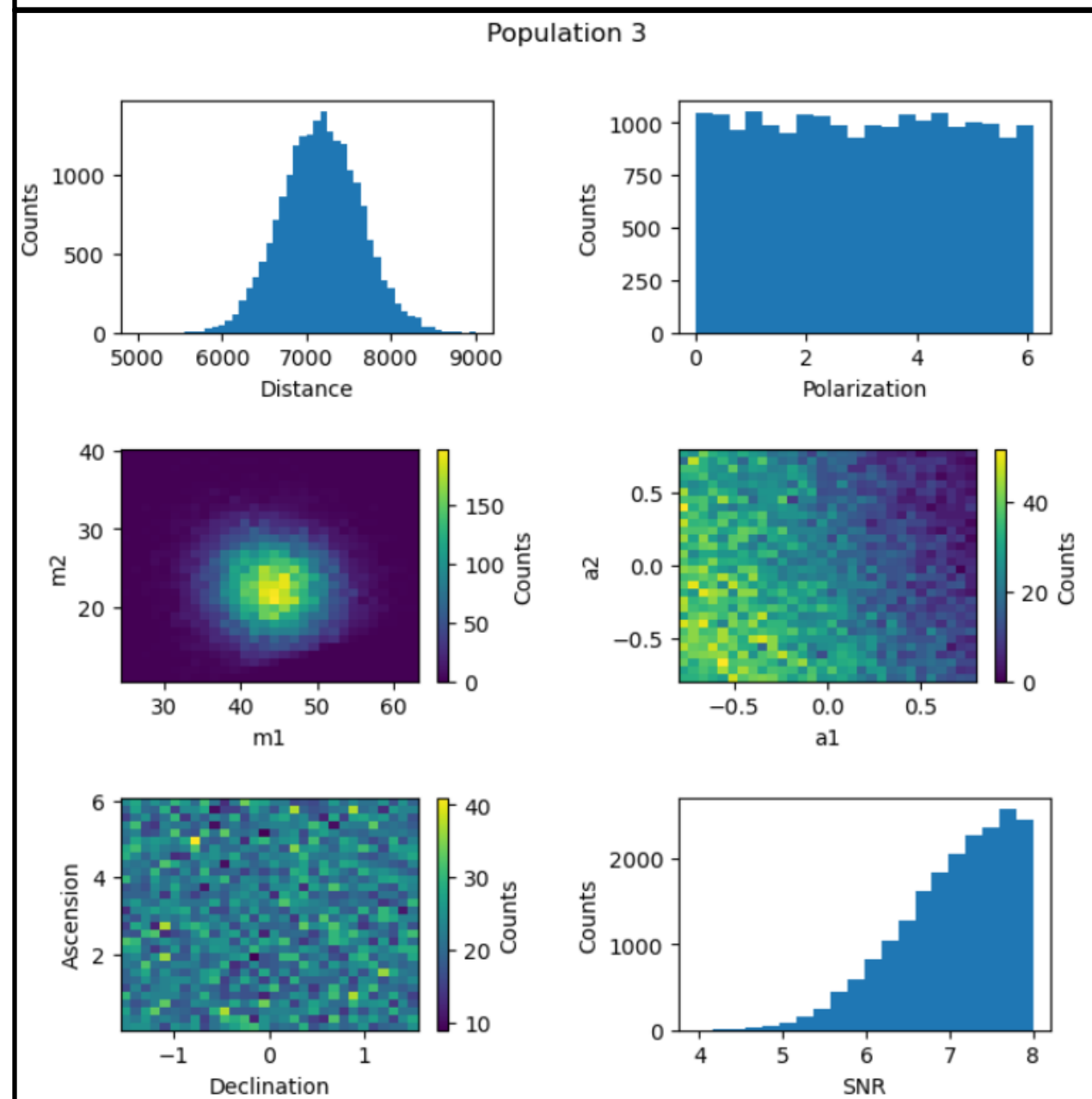
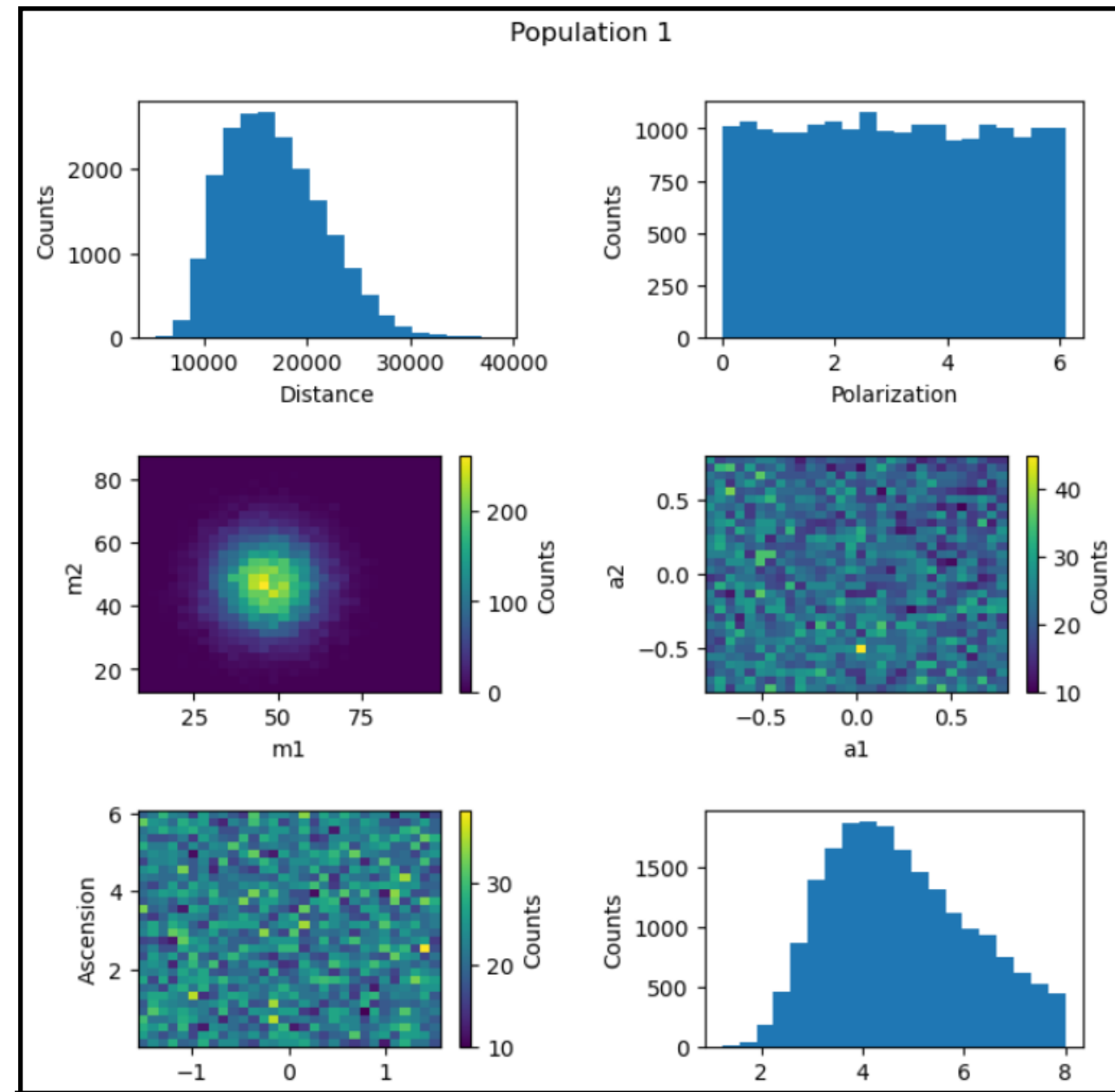
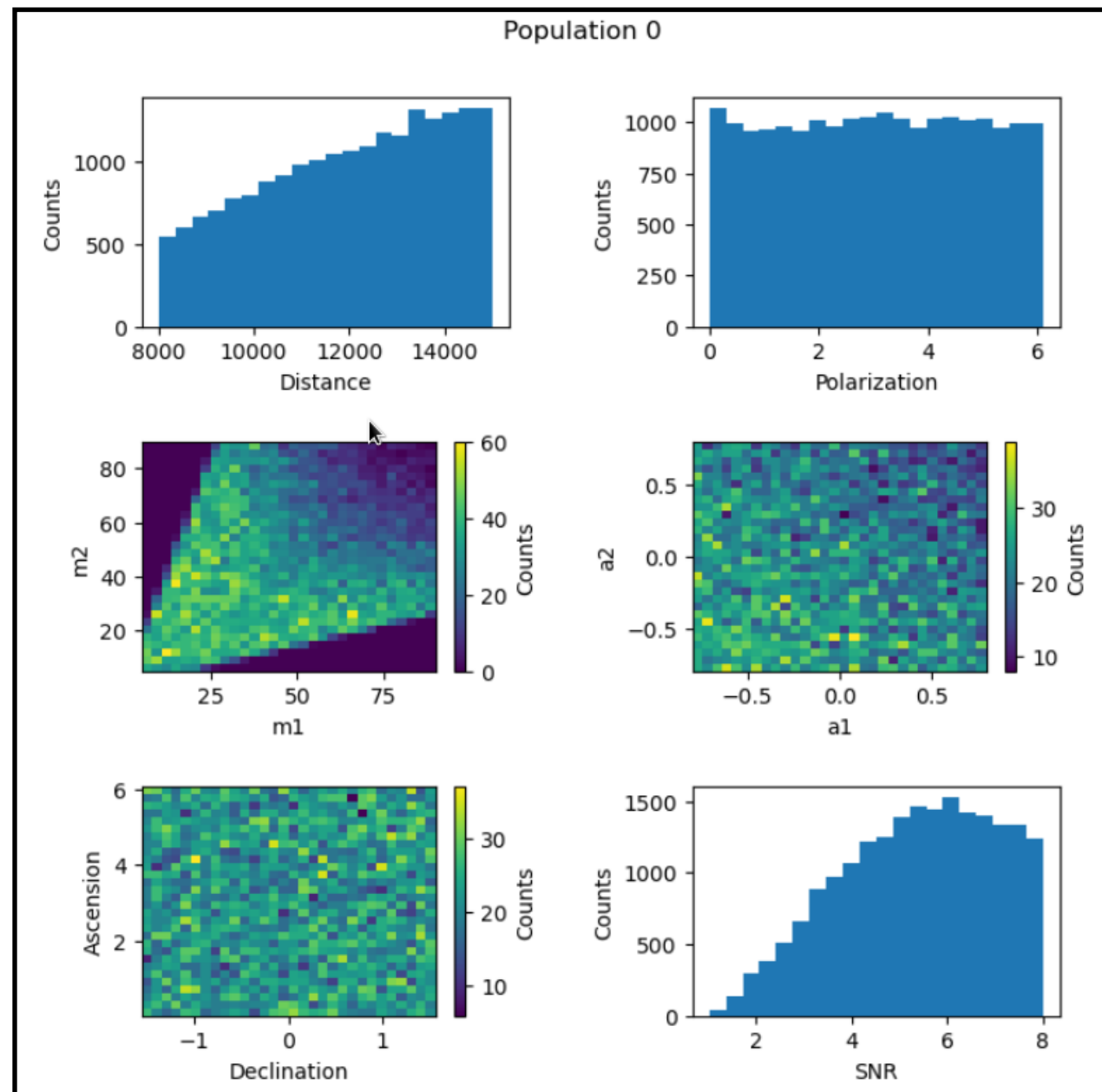


Inferring the SGW source population w/ SpeCs



Longo, Dey & NA, in prep.

Inferring the SGW source population w/ SpeCs

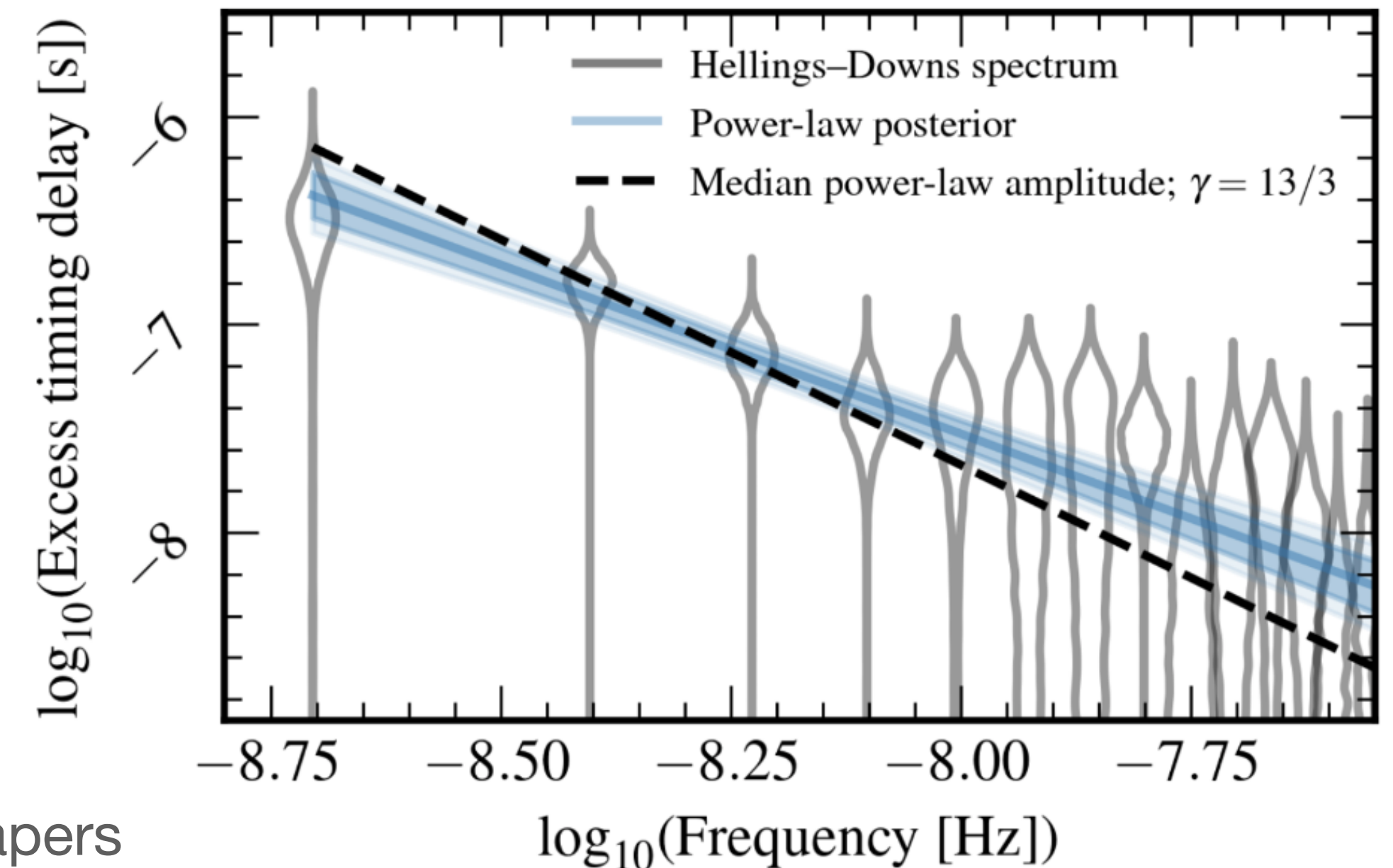
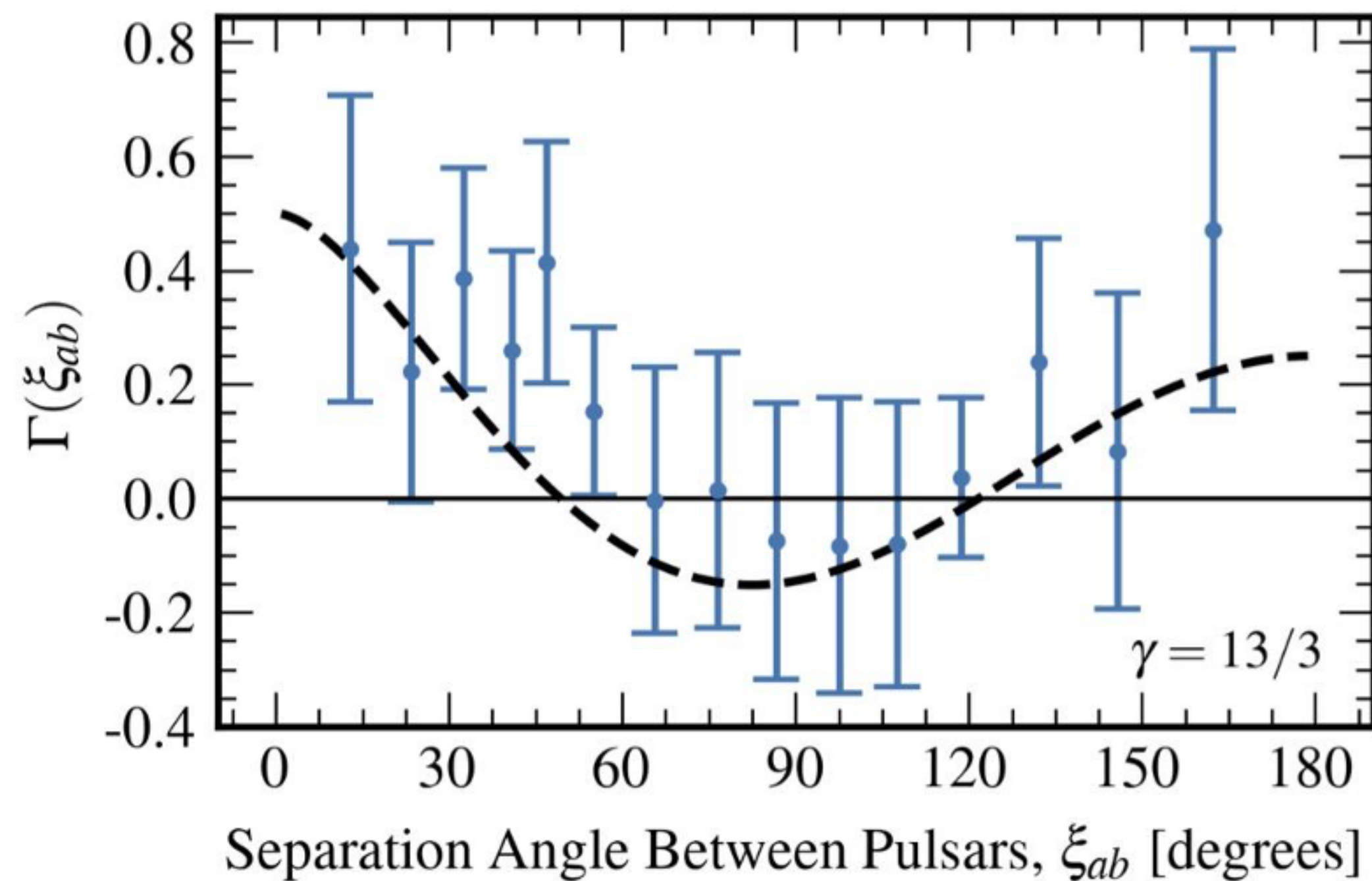
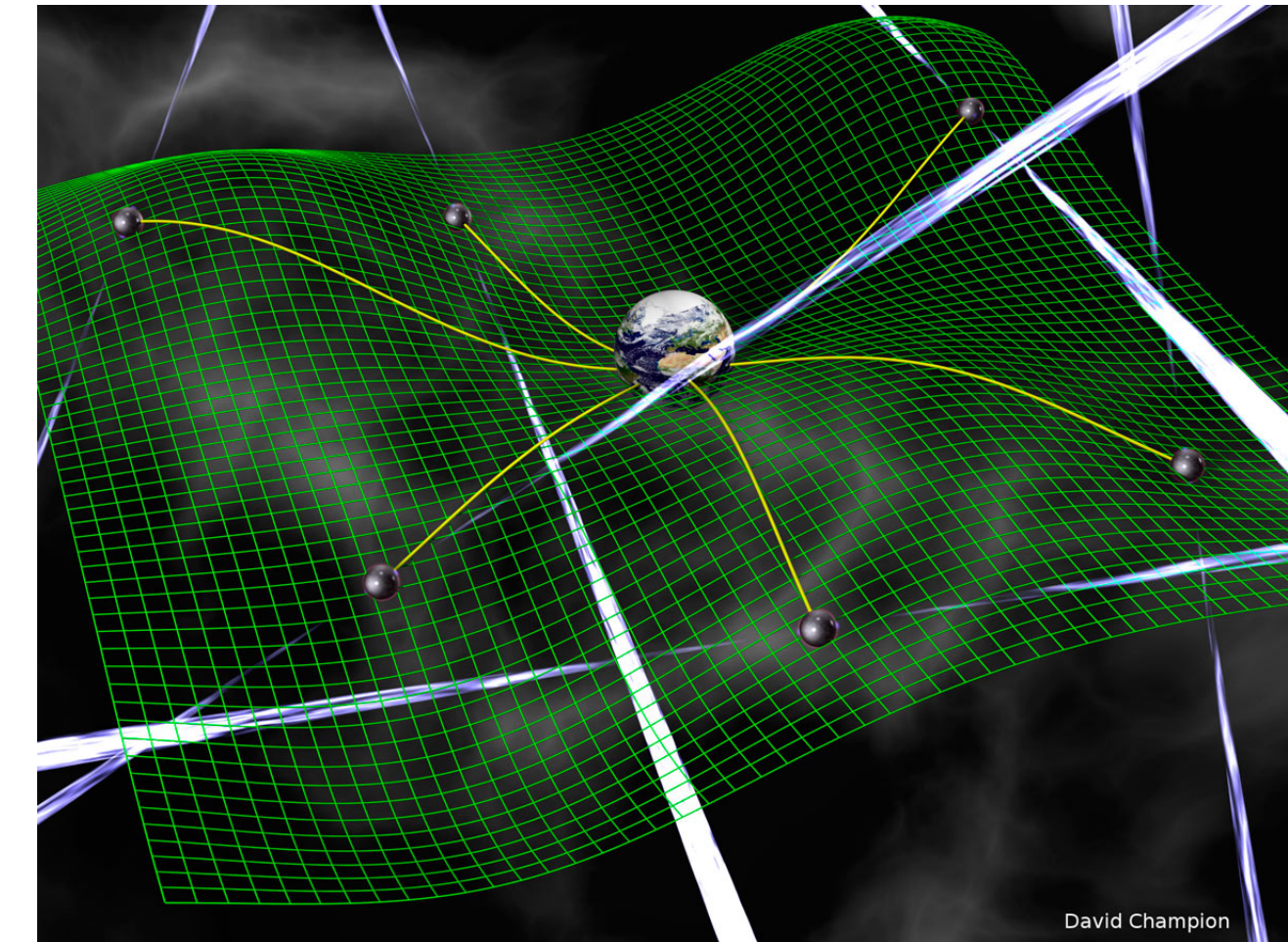


Longo, Dey & NA, in prep.



Can we apply the same idea to pulsar timing arrays?

Going beyond the 2-point statistics



Optimal SGW Estimator from Pulsar Timing

Poisson vs Gaussian statistics

Optimal SGW Estimator from Pulsar Timing

Poisson vs Gaussian statistics

- **Poisson:** Supermassive Binary Black Hole sources have discrete frequencies

Optimal SGW Estimator from Pulsar Timing

Poisson vs Gaussian statistics

- **Poisson:** Supermassive Binary Black Hole sources have discrete frequencies
- We can use this to write the exact likelihood for timing noise in frequency space:

Optimal SGW Estimator from Pulsar Timing

Poisson vs Gaussian statistics

- **Poisson:** Supermassive Binary Black Hole sources have discrete frequencies
- We can use this to write the exact likelihood for timing noise in frequency spac

$$\ln \mathcal{L} = \int df n_s(f) \left[\left\langle \exp \left(-\frac{1}{2} \sum_p \frac{\left| \delta t_p(f) - \hat{p}^i \hat{p}^j A_{ij} \left\{ \frac{1 - \exp[2\pi i f r_p (1 - \hat{p} \cdot \hat{s})]}{2\pi f (1 - \hat{p} \cdot \hat{s})} \right\} \right|^2}{\sigma_p^2(f)} \right) \right\rangle_{A_{ij}, \hat{s}} - 1 \right]$$

Optimal SGW Estimator from Pulsar Timing

Poisson vs Gaussian statistics

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Distribution of
SMBH freq's

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Distribution of
SMBH freq's

Observed timing
for pulsar p

Observed timing
for pulsar p

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Distribution of
SMBH freq's

Observed timing
for pulsar p

Pulsar direction

Observed timing
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Distribution of
SMBH freq's

Observed timing
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Pulsar direction

Observed timing
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SMBH direction

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Distribution of
SMBH freq's

Observed timing
for pulsar p

Pulsar direction

Observed timing
for pulsar p

GW amplitude

SMBH direction

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Distribution of
SMBH freq's

Observed timing
for pulsar p

Pulsar direction

Observational noise

Observed timing
for pulsar p

GW amplitude

SMBH direction

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Distribution of
SMBH freq's

Observed timing
for pulsar p

Pulsar direction

Observational noise

Observed timing
for pulsar p

GW amplitude

SMBH direction

- Reduces to Helling-Downs for large noise, $\sigma_p(f)$, but in general, includes all higher order statistics

Conclusions

- **Echoes and Images:** probe quantum structure of BH horizons, beyond EFT
- **SpeCS:** Stacking spectrograms, a novel way of detecting stochastic GW background and its underlying physics